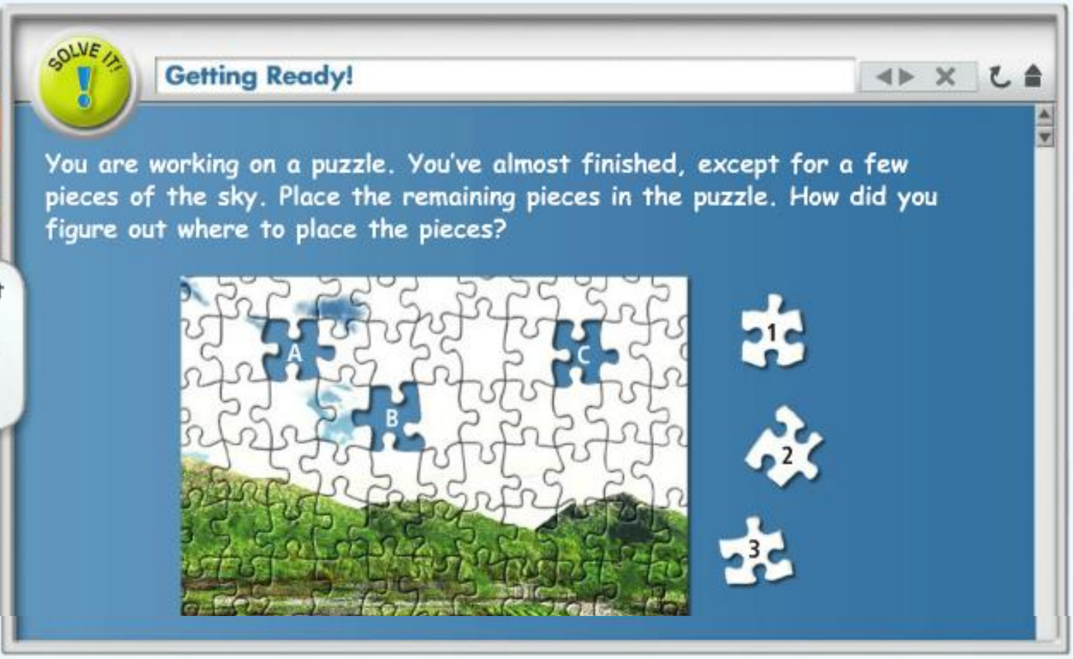
4-1

Congruent Figures

Objective To recognize congruent figures and their corresponding parts

They say you can't fit a square peg into a round hole. I wonder why that is.

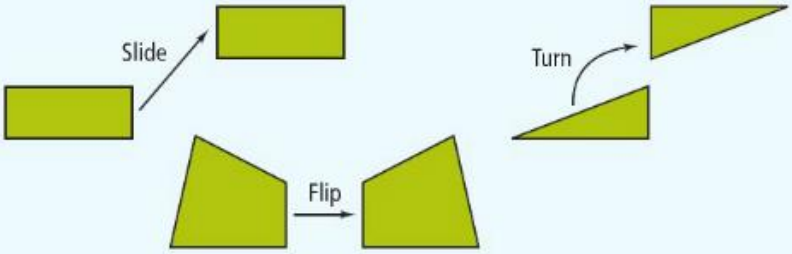


Getting Ready!

You are working on a puzzle. You've almost finished, except for a few pieces of the sky. Place the remaining pieces in the puzzle. How did you figure out where to place the pieces?

Lesson Vocabulary
- congruent polygons

Congruent figures have the same size and shape. When two figures are congruent, you can slide, flip, or turn one so that it fits exactly on the other one, as shown below. In this lesson, you will learn how to determine if geometric figures are congruent.



Essential Understanding You can determine whether two figures are congruent by comparing their corresponding parts.

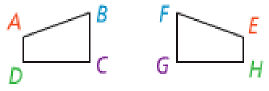
take note

Key Concept Congruent Figures

Definition

Congruent polygons have congruent corresponding parts—their matching sides and angles. When you name congruent polygons, you must list corresponding vertices in the same order.

Example



$ABCD \cong EFGH$

$\overline{AB} \cong \overline{EF}$ $\overline{BC} \cong \overline{FG}$
$\overline{CD} \cong \overline{GH}$ $\overline{DA} \cong \overline{HE}$

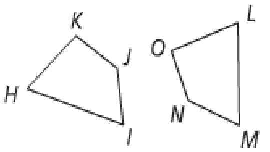
$\angle A \cong \angle E$ $\angle B \cong \angle F$
$\angle C \cong \angle G$ $\angle D \cong \angle H$

Plan

How do you know which sides and angles correspond?

The congruence statement $HIJK \cong LMNO$ tells you which parts correspond.

Problem 1 Finding Congruent Parts



If $HIJK \cong LMNO$, what are the congruent corresponding parts?

Sides: $\overline{HI} \cong \overline{LM}$ $\overline{IJ} \cong \overline{MN}$ $\overline{JK} \cong \overline{NO}$ $\overline{KH} \cong \overline{OL}$

Angles: $\angle H \cong \angle L$ $\angle I \cong \angle M$ $\angle J \cong \angle N$ $\angle K \cong \angle O$

Got It? **1.** If $\triangle WYS \cong \triangle MKV$, what are the congruent corresponding parts?

Plan

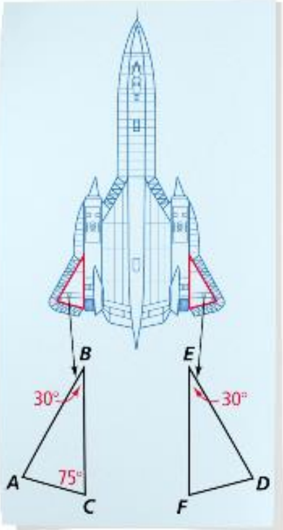
You know two angle measures in $\triangle ABC$. How can they help?

In the congruent triangles, $\angle D$ corresponds to $\angle A$, so you know that $\angle D \cong \angle A$. You can find $m\angle D$ by first finding $m\angle A$.

Problem 2 Using Congruent Parts

Multiple Choice **The wings of an SR-71 Blackbird aircraft suggest congruent triangles. What is $m\angle D$?**

Ⓐ 30 Ⓑ 75 Ⓒ 105 Ⓓ 150



Think	Write
Use the Triangle Angle-Sum Theorem to write an equation involving $m\angle A$.	$m\angle A + 30 + 75 = 180$
Solve for $m\angle A$.	$m\angle A + 105 = 180$ $m\angle A = 75$
$\angle A$ and $\angle D$ are corresponding parts of congruent triangles, so $\angle A \cong \angle D$.	$m\angle A = m\angle D = 75$ The correct answer is B.

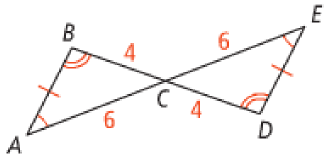
Got It? **2.** Suppose that $\triangle WYS \cong \triangle MKV$. If $m\angle W = 62$ and $m\angle Y = 35$, what is $m\angle V$? Explain.

Problem 3 Finding Congruent Triangles

Plan

How do you determine whether two triangles are congruent?
Compare each pair of corresponding parts. If all six pairs are congruent, then the triangles are congruent.

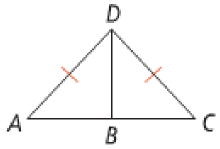
Are the triangles congruent? Justify your answer.



$\overline{AB} \cong \overline{ED}$	Given
$\overline{BC} \cong \overline{DC}$	$BC = 4 = DC$
$\overline{AC} \cong \overline{EC}$	$AC = 6 = EC$
$\angle A \cong \angle E$, $\angle B \cong \angle D$	Given
$\angle BCA \cong \angle DCE$	Vertical angles are congruent.

$\triangle ABC \cong \triangle EDC$ by the definition of congruent triangles.

Got It? **3.** Is $\triangle ABD \cong \triangle CBD$? Justify your answer.

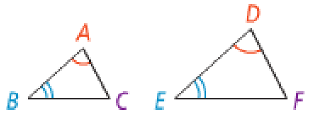


Recall the Triangle Angle-Sum Theorem: The sum of the measures of the angles in a triangle is 180. The next theorem follows from the Triangle Angle-Sum Theorem.

take note

Theorem 4-1 Third Angles Theorem

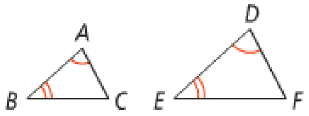
Theorem	If . . .	Then . . .
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.	$\angle A \cong \angle D$ and $\angle B \cong \angle E$	$\angle C \cong \angle F$



Proof Proof of Theorem 4-1: Third Angles Theorem

Given: $\angle A \cong \angle D$, $\angle B \cong \angle E$

Prove: $\angle C \cong \angle F$



Statements	Reasons
1) $\angle A \cong \angle D$, $\angle B \cong \angle E$	**1)** Given
2) $m\angle A = m\angle D$, $m\angle B = m\angle E$	**2)** Def. of $\cong$ ∠s
3) $m\angle A + m\angle B + m\angle C = 180$, $m\angle D + m\angle E + m\angle F = 180$	**3)** △ Angle-Sum Thm.
4) $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	**4)** Subst. Prop.
5) $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$	**5)** Subst. Prop.
6) $m\angle C = m\angle F$	**6)** Subtraction Prop. of =
7) $\angle C \cong \angle F$	**7)** Def. of $\cong$ ∠s

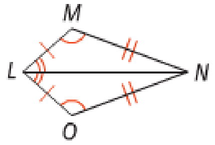
Proof Problem 4 Proving Triangles Congruent

Plan

You know four pairs of congruent parts. What else do you need to prove the triangles congruent? You need a third pair of congruent sides and a third pair of congruent angles.

Given: $\overline{LM} \cong \overline{LO}$, $\overline{MN} \cong \overline{ON}$,
$\angle M \cong \angle O$, $\angle MLN \cong \angle OLN$

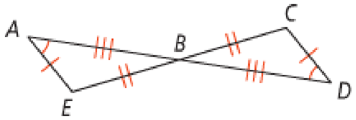
Prove: $\triangle LMN \cong \triangle LON$



Statements	Reasons
1) $\overline{LM} \cong \overline{LO}$, $\overline{MN} \cong \overline{ON}$	1) Given
2) $\overline{LN} \cong \overline{LN}$	2) Reflexive Property of $\cong$
3) $\angle M \cong \angle O$, $\angle MLN \cong \angle OLN$	3) Given
4) $\angle MNL \cong \angle ONL$	4) Third Angles Theorem
5) $\triangle LMN \cong \triangle LON$	5) Definition of $\cong$ triangles

Got It? **4. Given:** $\angle A \cong \angle D$, $\overline{AE} \cong \overline{DC}$,
$\overline{EB} \cong \overline{CB}$, $\overline{BA} \cong \overline{BD}$

Prove: $\triangle AEB \cong \triangle DCB$



Lesson Check

Do you know HOW?

Complete the following statements.

1. Given: $\triangle QXR \cong \triangle NYC$

a. $\overline{QX} \cong \underline{\ ?\ }$

b. $\angle Y \cong \underline{\ ?\ }$

2. Given: $\triangle BAT \cong \triangle FOR$

a. $\overline{TA} \cong \underline{\ ?\ }$

b. $\angle R \cong \underline{\ ?\ }$

3. Given: $BAND \cong LUCK$

a. $\angle U \cong \underline{\ ?\ }$

b. $\overline{DB} \cong \underline{\ ?\ }$

c. $NDBA \cong \underline{\ ?\ }$

4. In $\triangle MAP$ and $\triangle TIE$, $\angle A \cong \angle I$ and $\angle P \cong \angle E$.

a. What is the relationship between $\angle M$ and $\angle T$?

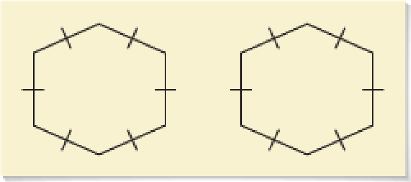
b. If $m\angle A = 52$ and $m\angle P = 36$, what is $m\angle T$?

Do you UNDERSTAND?

5. Open-Ended When do you think you might need to know that things are congruent in your everyday life?

6. If each angle in one triangle is congruent to its corresponding angle in another triangle, are the two triangles congruent? Explain.

7. Error Analysis Walter sketched the diagram below. He claims it shows that the two polygons are congruent. What information is missing to support his claim?

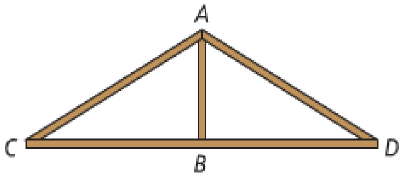


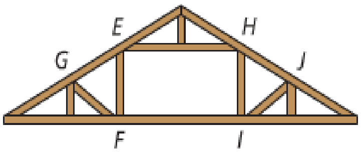
Practice and Problem-Solving Exercises

A Practice

8. **Construction** Builders use the king post truss (below left) for the top of a simple structure. In this truss, $\triangle ABC \cong \triangle ABD$. List the congruent corresponding parts.

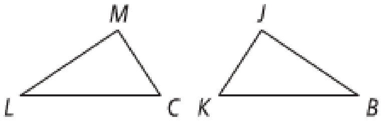
See Problem 1.





9. The attic frame truss (above right) provides open space in the center for storage. In this truss, $\triangle EFG \cong \triangle HIJ$. List the congruent corresponding parts.

$\triangle LMC \cong \triangle BJK$. Complete the congruence statements.



10. $\overline{LC} \cong \underline{\ ?\ }$

11. $\overline{KJ} \cong \underline{\ ?\ }$

12. $\overline{JB} \cong \underline{\ ?\ }$

13. $\angle L \cong \underline{\ ?\ }$

14. $\angle K \cong \underline{\ ?\ }$

15. $\angle M \cong \underline{\ ?\ }$

16. $\triangle CML \cong \underline{\ ?\ }$

17. $\triangle KBJ \cong \underline{\ ?\ }$

18. $\triangle MLC \cong \underline{\ ?\ }$

19. $\triangle JKB \cong \underline{\ ?\ }$

$POLY \cong SIDE$. List each of the following.

20. four pairs of congruent sides

21. four pairs of congruent angles

At an archeological site, the remains of two ancient step pyramids are congruent. If $ABCD \cong EFGH$, find each of the following. (Diagrams are not to scale.)

See Problem 2.

22. AD

23. GH

24. $m\angle GHE$

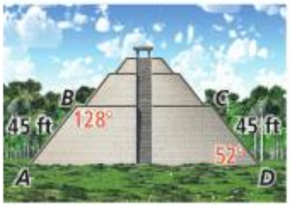
25. $m\angle BAD$

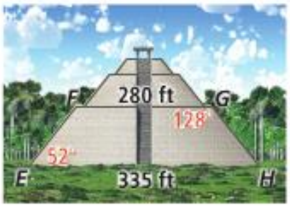
26. EF

27. BC

28. $m\angle DCB$

29. $m\angle EFG$

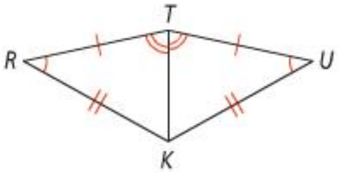




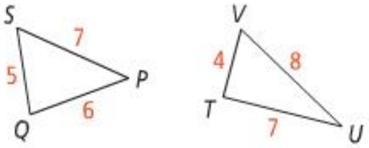
For Exercises 30 and 31, can you conclude that the triangles are congruent? Justify your answers.

See Problem 3.

30. $\triangle TRK$ and $\triangle TUK$

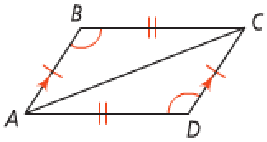


31. $\triangle SPQ$ and $\triangle TUV$



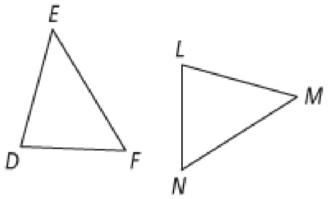
See Problem 4.

32. Given: $\overline{AB} \parallel \overline{DC}$, $\angle B \cong \angle D$,
$\overline{AB} \cong \overline{DC}$, $\overline{BC} \cong \overline{AD}$
Proof
Prove: $\triangle ABC \cong \triangle CDA$



B Apply

33. If $\triangle DEF \cong \triangle LMN$, which of the following must be a correct congruence statement?



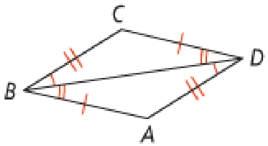
(A) $\overline{DE} \cong \overline{LN}$

(B) $\overline{FE} \cong \overline{NL}$

(C) $\angle N \cong \angle F$

(D) $\angle M \cong \angle F$

34. **Reasoning** Randall says he can use the information in the figure to prove $\triangle BCD \cong \triangle DAB$. Is he correct? Explain.



Algebra **$\triangle ABC \cong \triangle DEF$. Find the measures of the given angles or the lengths of the given sides.**

35. $m\angle A = x + 10$, $m\angle D = 2x$

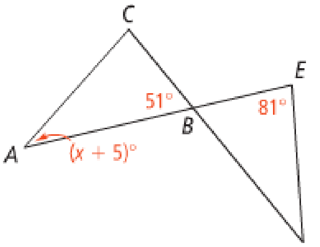
36. $m\angle B = 3y$, $m\angle E = 6y - 12$

37. $BC = 3z + 2$, $EF = z + 6$

38. $AC = 7a + 5$, $DF = 5a + 9$

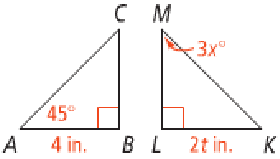
39. **Think About a Plan** $\triangle ABC \cong \triangle DBE$. Find the value of x.

- What does it mean for two triangles to be congruent?
- Which angle measures do you already know?
- How can you find the missing angle measure in a triangle?



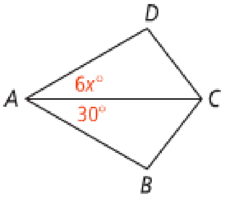
Algebra **Find the values of the variables.**

40.



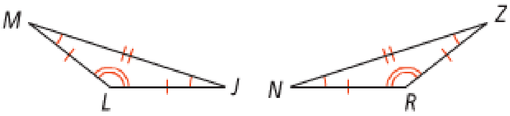
$\triangle ABC \cong \triangle KLM$

41.



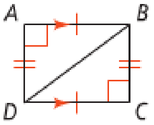
$\triangle ACD \cong \triangle ACB$

42. Complete in two different ways:
$\triangle JLM \cong$ __?__.



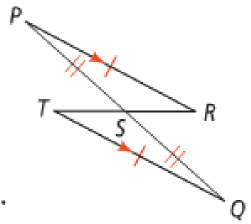
43. **Open-Ended** Write a congruence statement for two triangles. List the congruent sides and angles.

44. Given: $\overline{AB} \perp \overline{AD}$, $\overline{BC} \perp \overline{CD}$, $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$, $\overline{AB} \parallel \overline{CD}$
Proof
Prove: $\triangle ABD \cong \triangle CDB$



45. **Given:** $\overline{PR} \parallel \overline{TQ}$, $\overline{PR} \cong \overline{TQ}$, $\overline{PS} \cong \overline{QS}$, $\overline{PQ}$ bisects $\overline{RT}$

Proof **Prove:** $\triangle PRS \cong \triangle QTS$



46. **Writing** The 225 cards in Tracy's sports card collection are rectangles of three different sizes. How could Tracy quickly sort the cards?

Challenge

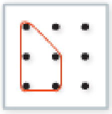
Coordinate Geometry **The vertices of $\triangle GHJ$ are $G(-2, -1)$, $H(-2, 3)$, and $J(1, 3)$.**

47. $\triangle KLM \cong \triangle GHJ$. Find KL, LM, and KM.

48. If L and M have coordinates $L(3, -3)$ and $M(6, -3)$, how many pairs of coordinates are possible for K? Find one such pair.

49. **a.** How many quadrilaterals (convex and concave) with different shapes or sizes can you make on a three-by-three geoboard? Sketch them. One is shown at the right.

b. How many quadrilaterals of each type are there?



Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

50. $\triangle HLN \cong \triangle GST$, $m\angle H = 66$, and $m\angle S = 42$. What is $m\angle T$?

51. The measure of one angle in a triangle is 80. The other two angles are congruent. What is the measure of each?

52. Given $A(2, -6)$ and $B(-3, 6)$, what is AB?

53. What is the number of feet in the perimeter of a square with side length 7 ft?

Mixed Review

Write an equation for the line perpendicular to the given line that contains P. **See Lesson 3-8.**

54. $P(2, 7)$; $y = \frac{3}{2}x - 2$

55. $P(1, 1)$; $y = 4x + 3$

Find the distance between the points. If necessary, round to the nearest tenth. **See Lesson 1-7.**

56. $A(0, 0)$, $B(4, 3)$

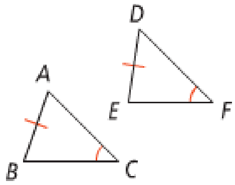
57. $X(11, 24)$, $Y(-7, 24)$

58. $E(1, -12)$, $F(1, -2)$

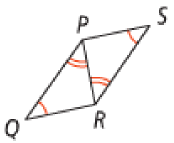
Get Ready! **To prepare for Lesson 4-2, do Exercises 59–61.**

What can you conclude from each diagram? **See Lessons 1-4 and 3-2.**

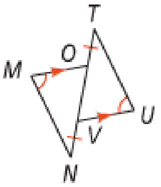
59.



60.



61.



Concept Byte

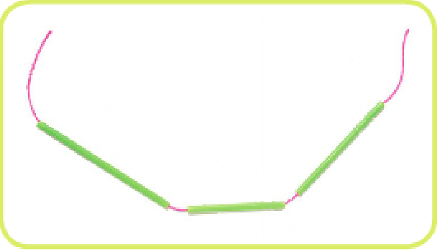
Use With Lesson 4-2

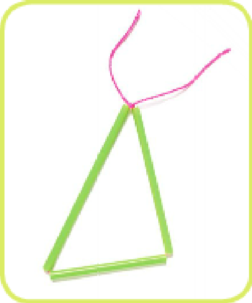
ACTIVITY

Building Congruent Triangles

Can you use shortcuts to find congruent triangles? Find out by building and comparing triangles.

Activity 1





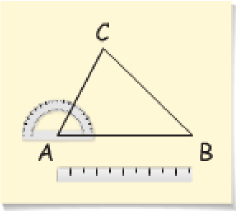
Step 1 Cut straws into three pieces of lengths 4 in., 5 in., and 6 in. Thread a string through the three pieces of straw. The straw pieces can be in any order.

Step 2 Bring the two ends of the string together to make a triangle. Tie the ends to hold your triangle in place.

Step 3 Compare your triangle with your classmates' triangles. Try to make your triangle fit exactly on top of the other triangles.

1. Is your triangle congruent to your classmates' triangles?
2. **Make a Conjecture** What seems to be true about two triangles in which three sides of one are congruent to three sides of another?
3. As a class, choose three different lengths and repeat Steps 1–3. Are all the triangles congruent? Does this support your conjecture from Question 2?

Activity 2



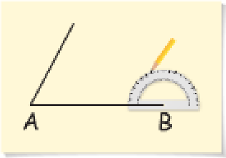
Step 1 Use a straightedge to draw and label any $\triangle ABC$ on tracing paper.

Step 2 Use a ruler. Carefully measure $\overline{AB}$ and $\overline{AC}$. Use a protractor to measure the angle between them, $\angle A$.

Step 3 Write the measurements on an index card and swap cards with a classmate. Draw a triangle using only your classmate's measurements.

Step 4 Compare your new triangle to your classmate's original $\triangle ABC$. Try to make your classmate's $\triangle ABC$ fit exactly on top of your new triangle.

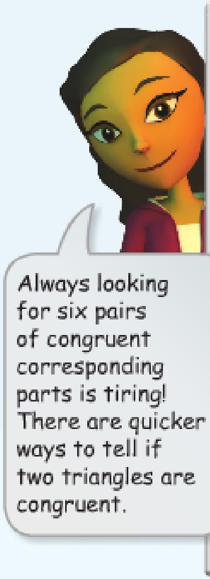
4. Is your new triangle congruent to your classmate's original $\triangle ABC$?
5. **Make a Conjecture** What seems to be true about two triangles when they have two congruent sides and a congruent angle between them?
6. **Make a Conjecture** At least how many triangle measurements must you know in order to guarantee that all triangles built with those measurements will be congruent?

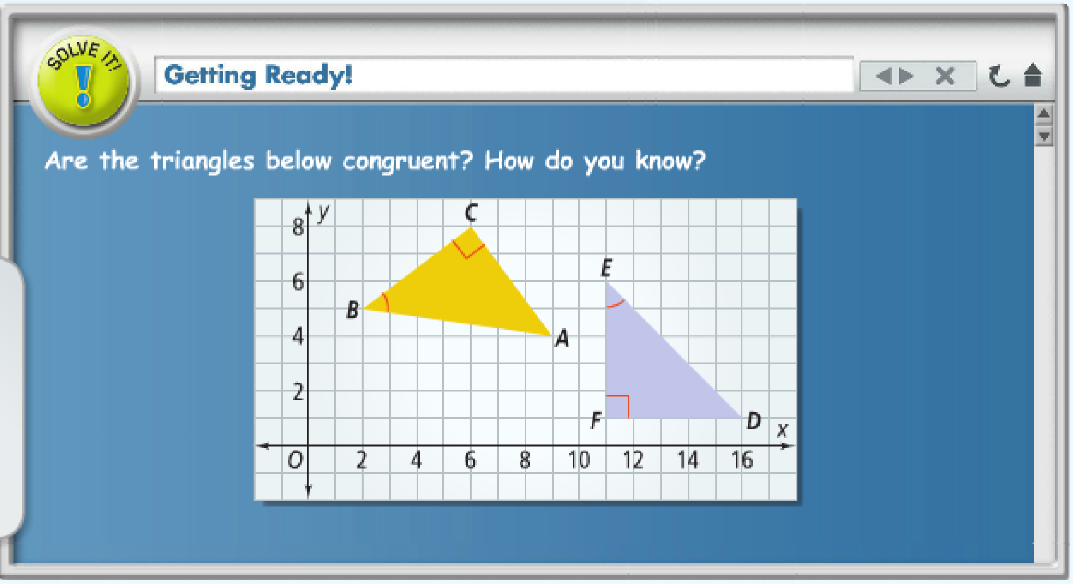


4-2

Triangle Congruence by SSS and SAS

Objective To prove two triangles congruent using the SSS and SAS Postulates


Always looking for six pairs of congruent corresponding parts is tiring! There are quicker ways to tell if two triangles are congruent.



Getting Ready!

Are the triangles below congruent? How do you know?

Dynamic Activity
Congruent Triangles

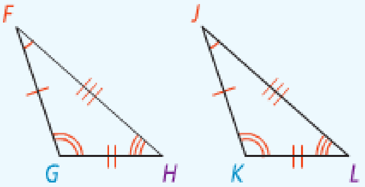
In the Solve It, you looked for relationships between corresponding sides and angles. In Lesson 4-1, you learned that if two triangles have three pairs of congruent corresponding angles and three pairs of congruent corresponding sides, then the triangles are congruent.

If you know . . .

$\angle F \cong \angle J$ $\overline{FG} \cong \overline{JK}$

$\angle G \cong \angle K$ $\overline{GH} \cong \overline{KL}$

$\angle H \cong \angle L$ $\overline{FH} \cong \overline{JL}$



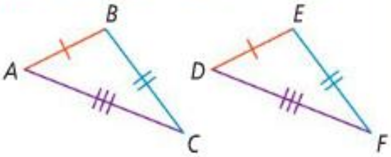
. . . then you know $\triangle FGH \cong \triangle JKL$.

However, this is more information about the corresponding parts than you need to prove triangles congruent.

Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove triangles congruent by using (1) three pairs of corresponding sides and (2) two pairs of corresponding sides and one pair of corresponding angles.

take note

Postulate 4-1 Side-Side-Side (SSS) Postulate

Postulate	If . . .	Then . . .
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.	$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$ 	$\triangle ABC \cong \triangle DEF$

As described in Chapter 1, a postulate is an accepted statement of fact. The Side-Side-Side Postulate is perhaps the most logical fact about triangles. It agrees with the notion that triangles are rigid figures; their shape does not change until pressure on their sides forces them to break. This rigidity property is important to architects and engineers when they build things such as bicycle frames and steel bridges.

Proof

Problem 1 Using SSS

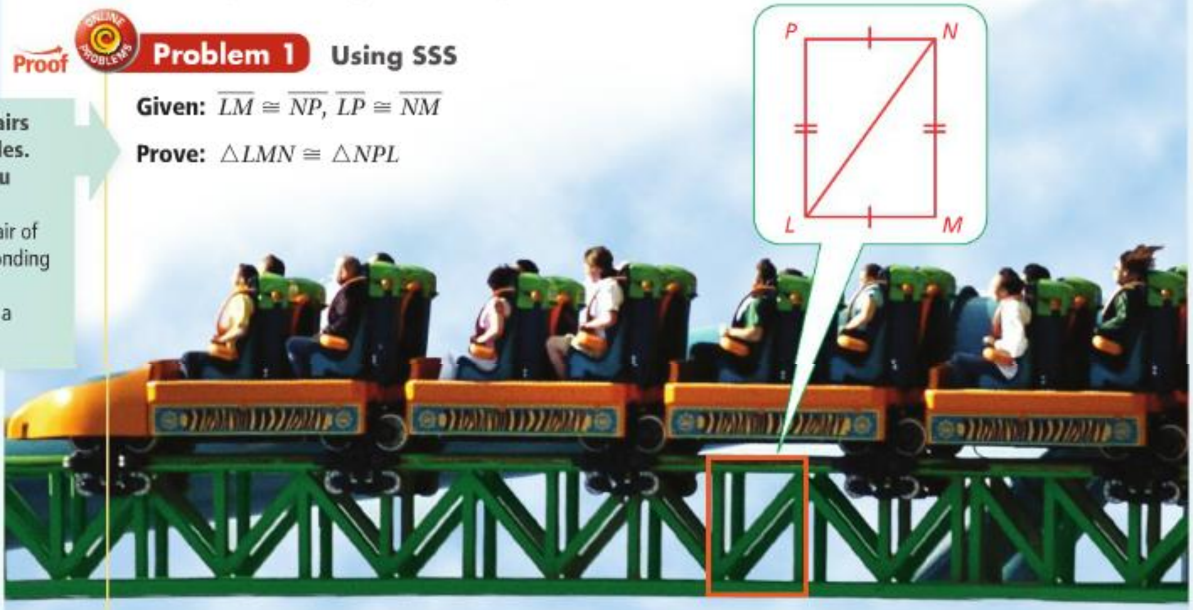
Plan

You have two pairs of congruent sides. What else do you need?

You need a third pair of congruent corresponding sides. Notice that the triangles share a common side, $\overline{LN}$.

Given: $\overline{LM} \cong \overline{NP}$, $\overline{LP} \cong \overline{NM}$

Prove: $\triangle LMN \cong \triangle NPL$



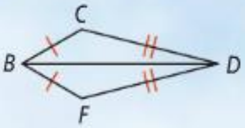
$\overline{LM} \cong \overline{NP}$
Given

$\overline{LN} \cong \overline{LN}$
Reflexive Prop. of ≅

$\overline{LP} \cong \overline{NM}$
Given

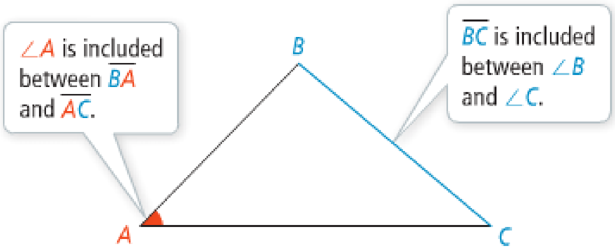
$\triangle LMN \cong \triangle NPL$
SSS

Got It? **1. Given:** $\overline{BC} \cong \overline{BF}$, $\overline{CD} \cong \overline{FD}$
Prove: $\triangle BCD \cong \triangle BFD$



You can also show relationships between a pair of corresponding sides and an *included* angle.

The word *included* refers to the angles and the sides of a triangle as shown at the right.



$\angle A$ is included between $\overline{BA}$ and $\overline{AC}$.

$\overline{BC}$ is included between $\angle B$ and $\angle C$.

take note

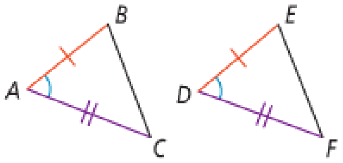
Postulate 4-2 Side-Angle-Side (SAS) Postulate

Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

If . . .

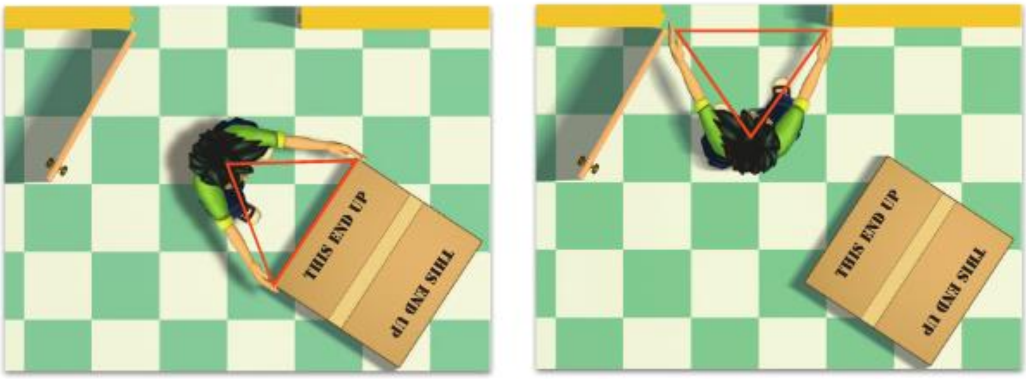
$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$



Then . . .

$\triangle ABC \cong \triangle DEF$

You likely have used the properties of the Side-Angle-Side Postulate before. For example, SAS can help you determine whether a box will fit through a doorway.



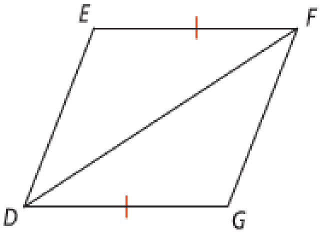
Suppose you keep your arms at a fixed angle as you move from the box to the doorway. The triangle you form with the box is congruent to the triangle you form with the doorway. The two triangles are congruent because two sides and the included angle of one triangle are congruent to the two sides and the included angle of the other triangle.

Problem 2 Using SAS

Plan

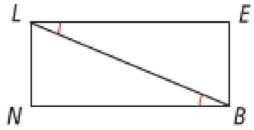
Do you need another pair of congruent sides?
Look at the diagram. The triangles share $\overline{DF}$. So you already have two pairs of congruent sides.

What other information do you need to prove $\triangle DEF \cong \triangle FGD$ by SAS? Explain.

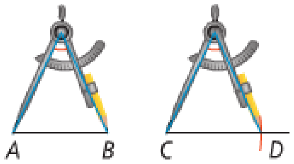


The diagram shows that $\overline{EF} \cong \overline{GD}$. Also, $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of Congruence. To prove that $\triangle DEF \cong \triangle FGD$ by SAS, you must have congruent included angles. You need to know that $\angle EFD \cong \angle GDF$.

Got It? **2.** What other information do you need to prove $\triangle LEB \cong \triangle BNL$ by SAS?



Recall that, in Lesson 1-6, you learned to construct segments using a compass open to a fixed angle. Now you can show that it works. Similar to the situation with the box and the doorway, the Side-Angle-Side Postulate tells you that the triangles outlined at the right are congruent. So, $\overline{AB} \cong \overline{CD}$.



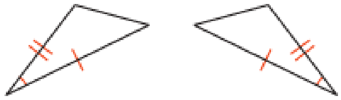
Problem 3 Identifying Congruent Triangles

Plan

What should you look for first, sides or angles?
Start with sides. If you have three pairs of congruent sides, use SSS. If you have two pairs of congruent sides, look for a pair of congruent included angles.

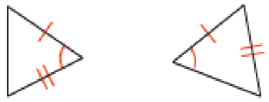
Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information*. Explain your answer.

A



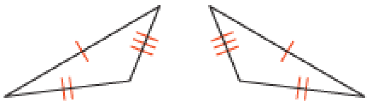
Use SAS because two pairs of corresponding sides and their included angles are congruent.

B



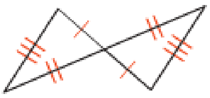
There is not enough information; two pairs of corresponding sides are congruent, but one of the angles is not the included angle.

C



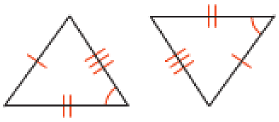
Use SSS because three pairs of corresponding sides are congruent.

D



Use SSS or SAS because all three pairs of corresponding sides and a pair of included angles (the vertical angles) are congruent.

Got It? **3.** Would you use SSS or SAS to prove the triangles at the right congruent? Explain.



Lesson Check

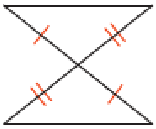
Do you know HOW?

1. In $\triangle PEN$, name the angle that is included between the given sides.
   a. $\overline{PE}$ and $\overline{EN}$ **b.** $\overline{NP}$ and $\overline{PE}$

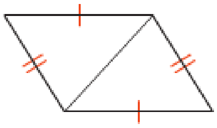
2. In $\triangle HAT$, between which sides is the given angle included?
   a. $\angle H$ **b.** $\angle T$

Name the postulate you would use to prove the triangles congruent.

3.



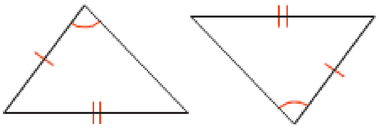
4.



Do you UNDERSTAND?

5. **Compare and Contrast** How are the SSS Postulate and the SAS Postulate alike? How are they different?

6. **Error Analysis** Your friend thinks that the triangles shown below are congruent by SAS. Is your friend correct? Explain.



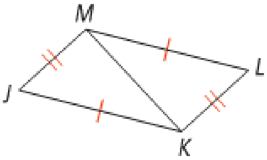
7. **Reasoning** A carpenter trims a triangular peak of a house with three 7-ft pieces of molding. The carpenter uses 21 ft of molding to trim a second triangular peak. Are the two triangles formed congruent? Explain.

Practice and Problem-Solving Exercises

A Practice

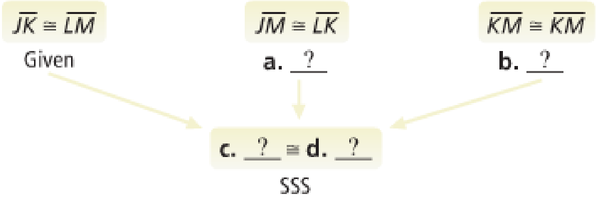
See Problem 1.

8. **Developing Proof** Copy and complete the flow proof.



Given: $\overline{JK} \cong \overline{LM}$, $\overline{JM} \cong \overline{LK}$

Prove: $\triangle JKM \cong \triangle LMK$



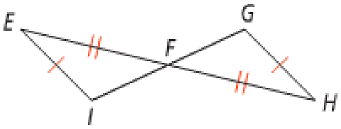
$\overline{JK} \cong \overline{LM}$
Given

$\overline{JM} \cong \overline{LK}$
a. ___?___

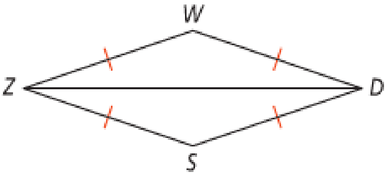
$\overline{KM} \cong \overline{KM}$
b. ___?___

c. ___?___ $\cong$ **d.** ___?___
SSS

9. **Given:** $\overline{IE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HF}$,
Proof F is the midpoint of $\overline{GI}$
Prove: $\triangle EFI \cong \triangle HFG$



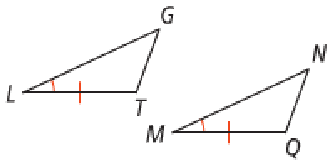
10. **Given:** $\overline{WZ} \cong \overline{ZS} \cong \overline{SD} \cong \overline{DW}$
Proof **Prove:** $\triangle WZD \cong \triangle SDZ$



What other information, if any, do you need to prove the two triangles congruent by SAS? Explain.

See Problem 2.

11.



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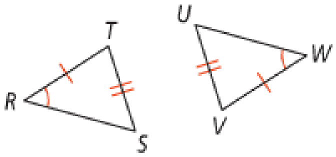
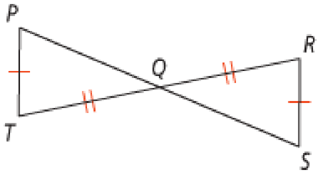


Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information*. Explain your answer.

See Problem 3.

13.



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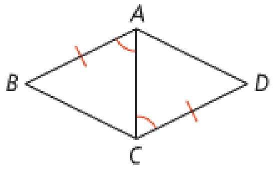


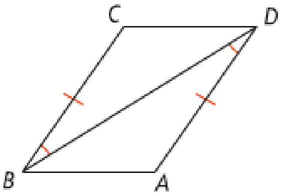
B Apply

15. Think About a Plan You and a friend are cutting triangles out of felt for an art project. You want all the triangles to be congruent. Your friend tells you that each triangle should have two 5-in. sides and a 40° angle. If you follow this rule, will all your felt triangles be congruent? Explain.

- How can you use diagrams to help you?
- Which postulate, SSS or SAS, are you likely to apply to the given situation?

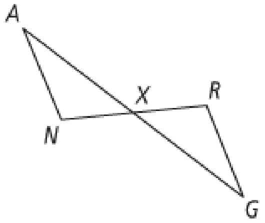
16. Given: $\overline{BC} \cong \overline{DA}$, $\angle CBD \cong \angle ADB$

Proof **Prove:** $\triangle BCD \cong \triangle DAB$



17. Given: X is the midpoint of $\overline{AG}$ and $\overline{NR}$.

Proof **Prove:** $\triangle ANX \cong \triangle GRX$



Use the Distance Formula to determine whether $\triangle ABC$ and $\triangle DEF$ are congruent. Justify your answer.

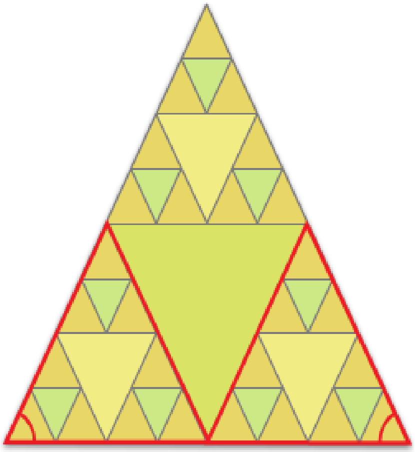
18. $A(1, 4)$, $B(5, 5)$, $C(2, 2)$; $D(-5, 1)$, $E(-1, 0)$, $F(-4, 3)$

19. $A(3, 8)$, $B(8, 12)$, $C(10, 5)$; $D(3, -1)$, $E(7, -7)$, $F(12, -2)$

20. $A(2, 9)$, $B(2, 4)$, $C(5, 4)$; $D(1, -3)$, $E(1, 2)$, $F(-2, 2)$

21. Writing List three real-life uses of congruent triangles. For each real-life use, describe why you think congruence is necessary.

22. Sierpinski's Triangle Sierpinski's triangle is a famous geometric pattern. To draw Sierpinski's triangle, start with a single triangle and connect the midpoints of the sides to draw a smaller triangle. If you repeat this pattern over and over, you will form a figure like the one shown. This particular figure started with an isosceles triangle. Are the triangles outlined in red congruent? Explain.

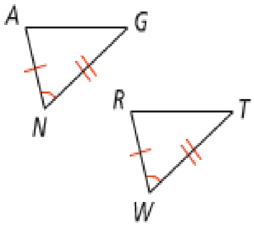


23. Constructions Use a straightedge to draw any triangle JKL. Then construct $\triangle MNP \cong \triangle JKL$ using the given postulate.

a. SSS
b. SAS

Can you prove the triangles congruent? If so, write the congruence statement and name the postulate you would use. If not, write *not enough information* and tell what other information you would need.

24.



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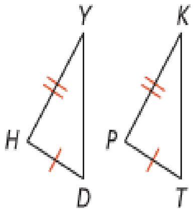

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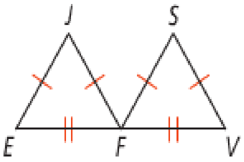
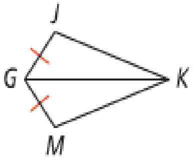


27. Reasoning Suppose $\overline{GH} \cong \overline{JK}$, $\overline{HI} \cong \overline{KL}$, and $\angle I \cong \angle L$. Is $\triangle GHI$ congruent to $\triangle JKL$? Explain.

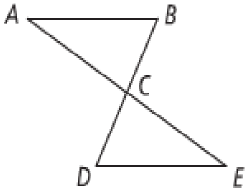
28. Given: $\overline{GK}$ bisects $\angle JGM$, $\overline{GJ} \cong \overline{GM}$

Proof **Prove:** $\triangle GJK \cong \triangle GMK$



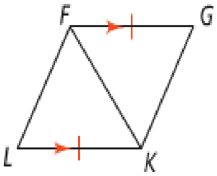
29. Given: $\overline{AE}$ and $\overline{BD}$ bisect each other.

Proof **Prove:** $\triangle ACB \cong \triangle ECD$



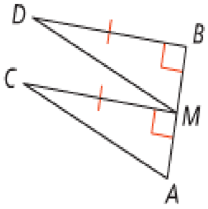
30. Given: $\overline{FG} \parallel \overline{KL}$, $\overline{FG} \cong \overline{KL}$

Proof **Prove:** $\triangle FGK \cong \triangle KLF$



31. Given: $\overline{AB} \perp \overline{CM}$, $\overline{AB} \perp \overline{DB}$, $\overline{CM} \cong \overline{DB}$, M is the midpoint of $\overline{AB}$

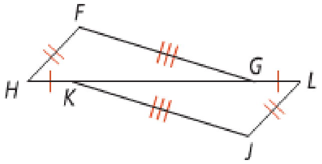
Proof **Prove:** $\triangle AMC \cong \triangle MBD$



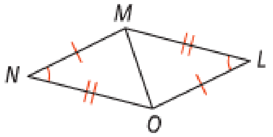
Challenge

32. **Reasoning** Four sides of polygon $ABCD$ are congruent, respectively, to the four sides of polygon $EFGH$. Are $ABCD$ and $EFGH$ congruent? Is a quadrilateral a rigid figure? If not, what could you add to make it a rigid figure? Explain.

33. **Given:** $\overline{HK} \cong \overline{LG}$, $\overline{HF} \cong \overline{LJ}$, $\overline{FG} \cong \overline{JK}$
Proof **Prove:** $\triangle FGH \cong \triangle JKL$



34. **Given:** $\angle N \cong \angle L$, $\overline{MN} \cong \overline{OL}$, $\overline{NO} \cong \overline{LM}$
Proof **Prove:** $\overline{MN} \parallel \overline{OL}$

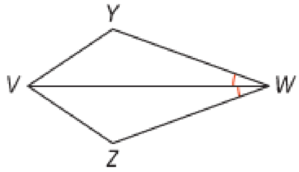


Standardized Test Prep

SAT/ACT

35. What additional information do you need to prove that $\triangle VWY \cong \triangle VWZ$ by SAS?

(A) $\overline{YW} \cong \overline{ZW}$
(B) $\angle WVY \cong \angle WVZ$
(C) $\angle Y \cong \angle Z$
(D) $\overline{VZ} \cong \overline{VY}$



36. The measures of two angles of a triangle are 43 and 38. What is the measure of the third angle?

(F) 9
(G) 81
(H) 99
(I) 100

37. Which method would you use to find the inverse of a conditional statement?

(A) Switch the hypothesis and conclusion.
(B) Negate the hypothesis only.
(C) Negate the conclusion only.
(D) Negate both the hypothesis and conclusion.

Short Response

38. A segment has a midpoint at $(1, 1)$ and an endpoint at $(-3, 4)$. What are the coordinates of the other endpoint of the segment? Show your work.

Mixed Review

$ABCD \cong EFGH$. Name the angle or side that corresponds to each part. **See Lesson 4-1.**

39. $\angle A$
40. $\overline{EF}$
41. $\overline{BC}$
42. $\angle G$

Write the converse of each statement. Determine whether the statement and its converse are true or false. **See Lesson 2-2.**

43. If $x = 3$, then $2x = 6$.
44. If $x = 3$, then $x^2 = 9$.

Get Ready! **To prepare for Lesson 4-3, do Exercises 45 and 46.**

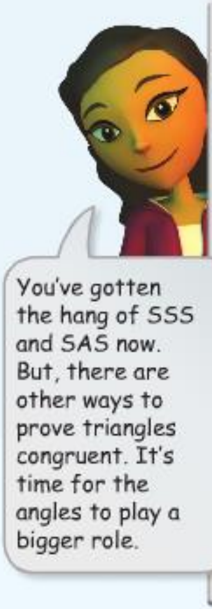
45. In $\triangle JHK$, name the side that is included between $\angle J$ and $\angle H$. **See Lesson 4-2.**

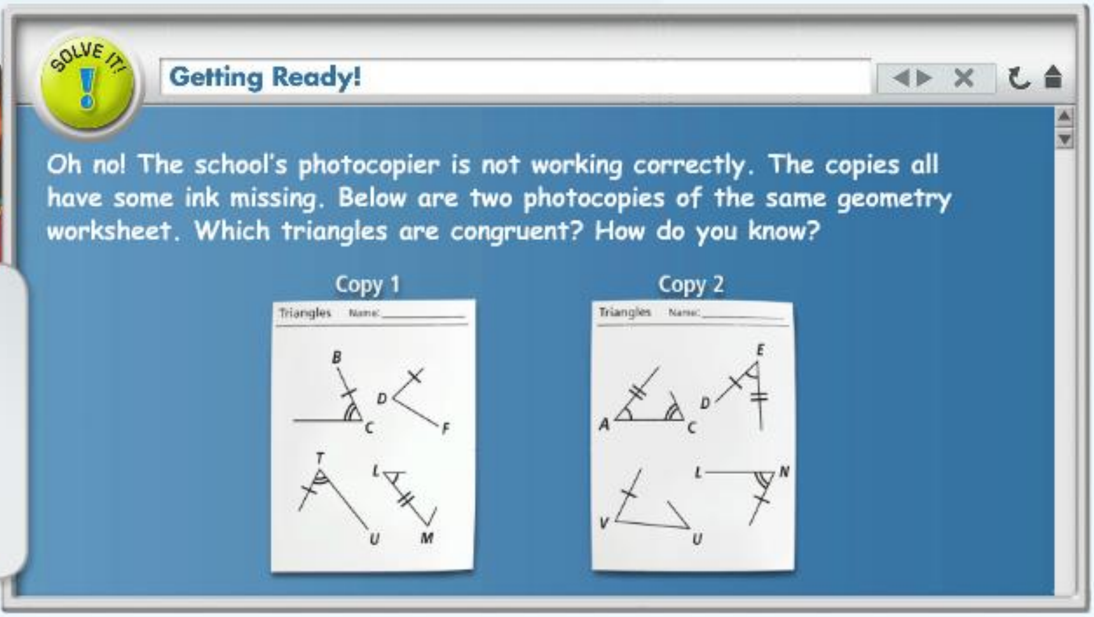
46. In $\triangle NLM$, name the angle that is included between $\overline{NM}$ and $\overline{LN}$.

4-3

Triangle Congruence by ASA and AAS

Objective To prove two triangles congruent using the ASA Postulate and the AAS Theorem


You've gotten the hang of SSS and SAS now. But, there are other ways to prove triangles congruent. It's time for the angles to play a bigger role.



Oh no! The school's photocopier is not working correctly. The copies all have some ink missing. Below are two photocopies of the same geometry worksheet. Which triangles are congruent? How do you know?

You already know that triangles are congruent if two pairs of sides and the included angles are congruent (SAS). You can also prove triangles congruent using other groupings of angles and sides.

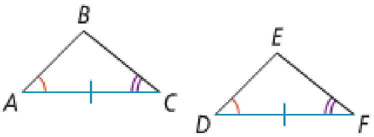
Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove triangles congruent by using one pair of corresponding sides and two pairs of corresponding angles.

take note

Postulate 4-3 Angle-Side-Angle (ASA) Postulate

Postulate
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

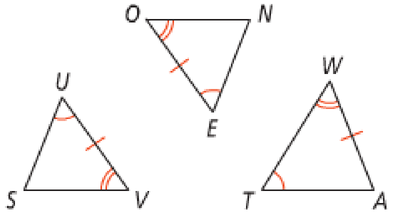
If . . .
$\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, $\angle C \cong \angle F$



Then . . .
$\triangle ABC \cong \triangle DEF$

Problem 1 Using ASA

Which two triangles are congruent by ASA? Explain.



Know

From the diagram you know

- $\angle U \cong \angle E \cong \angle T$
- $\angle V \cong \angle O \cong \angle W$
- $\overline{UV} \cong \overline{EO} \cong \overline{AW}$

Need

To use ASA, you need two pairs of congruent angles and a pair of included congruent sides.

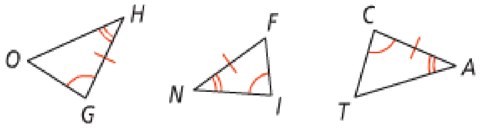
Plan

You already have pairs of congruent angles. So, identify the included side for each triangle and see whether it has a congruence marking.

In $\triangle SUV$, $\overline{UV}$ is included between $\angle U$ and $\angle V$ and has a congruence marking. In $\triangle NEO$, $\overline{EO}$ is included between $\angle E$ and $\angle O$ and has a congruence marking. In $\triangle ATW$, $\overline{TW}$ is included between $\angle T$ and $\angle W$ but does *not* have a congruence marking.

Since $\angle U \cong \angle E$, $\overline{UV} \cong \overline{EO}$, and $\angle V \cong \angle O$, $\triangle SUV \cong \triangle NEO$.

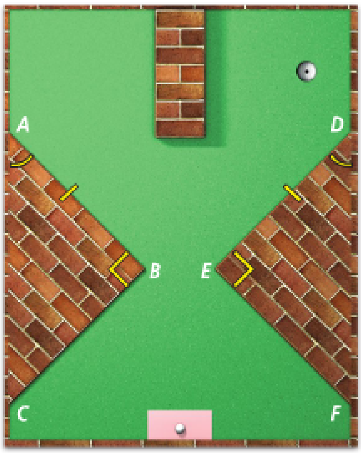
Got It? **1.** Which two triangles are congruent by ASA? Explain.



Problem 2 Writing a Proof Using ASA

Proof

Recreation Members of a teen organization are building a miniature golf course at your town's youth center. The design plan calls for the first hole to have two congruent triangular bumpers. Prove that the bumpers on the first hole, shown at the right, meet the conditions of the plan.



Given: $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\angle B$ and $\angle E$ are right angles

Prove: $\triangle ABC \cong \triangle DEF$

Proof: $\angle B \cong \angle E$ because all right angles are congruent, and you are given that $\angle A \cong \angle D$. $\overline{AB}$ and $\overline{DE}$ are included sides between the two pairs of congruent angles. You are given that $\overline{AB} \cong \overline{DE}$. Thus, $\triangle ABC \cong \triangle DEF$ by ASA.

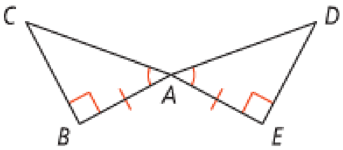
Plan

Can you use a plan similar to the plan in Problem 1?

Yes. Use the diagram to identify the included side for the marked angles in each triangle.

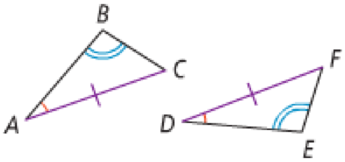
Got It? 2. Given: $\angle CAB \cong \angle DAE$, $\overline{BA} \cong \overline{EA}$,
$\angle B$ and $\angle E$ are right angles

Prove: $\triangle ABC \cong \triangle AED$



You can also prove triangles congruent by using two angles and a nonincluded side, as stated in the theorem below.

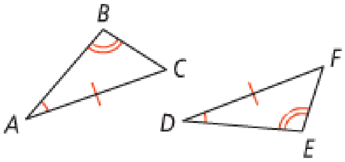
Theorem 4-2 Angle-Angle-Side (AAS) Theorem

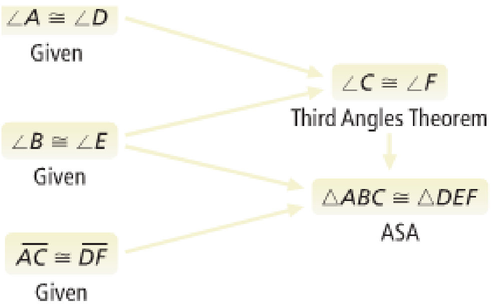
Theorem	**If . . .**	**Then . . .**
If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.	$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$ 	$\triangle ABC \cong \triangle DEF$

Proof Proof of Theorem 4-2: Angle-Angle-Side Theorem

Given: $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$

Prove: $\triangle ABC \cong \triangle DEF$





You have seen and used three methods of proof in this book—two-column, paragraph, and flow proof. Each method is equally as valid as the others. Unless told otherwise, you can choose any of the three methods to write a proof. Just be sure your proof always presents logical reasoning with justification.

Plan

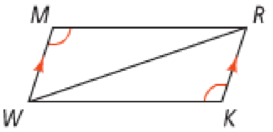
How does information about parallel sides help?
You will need another pair of congruent angles to use AAS. Think back to what you learned in Chapter 3. $\overline{WR}$ is a transversal here.

Proof

Problem 3 Writing a Proof Using AAS

Given: $\angle M \cong \angle K$, $\overline{WM} \parallel \overline{RK}$

Prove: $\triangle WMR \cong \triangle RKW$

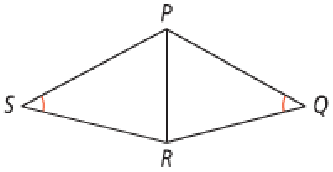


Statements	Reasons
1) $\angle M \cong \angle K$	**1)** Given
2) $\overline{WM} \parallel \overline{RK}$	**2)** Given
3) $\angle MWR \cong \angle KRW$	**3)** If lines are $\parallel$, then alternate interior $\measuredangle$ are $\cong$.
4) $\overline{WR} \cong \overline{WR}$	**4)** Reflexive Property of Congruence
5) $\triangle WMR \cong \triangle RKW$	**5)** AAS

Got It? **3. a. Given:** $\angle S \cong \angle Q$, $\overline{RP}$ bisects $\angle SRQ$

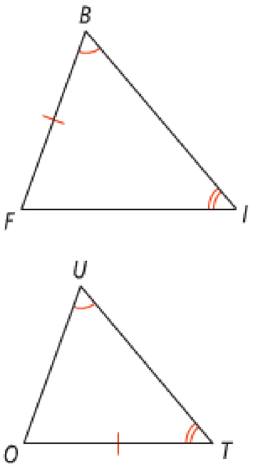
Prove: $\triangle SRP \cong \triangle QRP$

b. Reasoning In Problem 3, how could you prove that $\triangle WMR \cong \triangle RKW$ by ASA? Explain.



Problem 4 Determining Whether Triangles Are Congruent

Multiple Choice **Use the diagram at the right. Which of the following statements best represents the answer and justification to the question, "Is $\triangle BIF \cong \triangle UTO$?"**



Ⓐ Yes, the triangles are congruent by ASA.

Ⓑ No, $\overline{FB}$ and $\overline{OT}$ are not corresponding sides.

Ⓒ Yes, the triangles are congruent by AAS.

Ⓓ No, $\angle B$ and $\angle U$ are not corresponding angles.

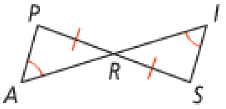
Think

Can you eliminate any of the choices?
Yes. If $\triangle BIF \cong \triangle UTO$ then $\angle B$ and $\angle U$ would be corresponding angles. You can eliminate choice D.

The diagram shows that two pairs of angles and one pair of sides are congruent. The third pair of angles are congruent by the Third Angles Theorem. To prove these triangles congruent, you need to satisfy ASA or AAS.

ASA and AAS both fail because $\overline{FB}$ and $\overline{TO}$ are not included between the same pair of congruent corresponding angles, so they are not corresponding sides. The triangles are not necessarily congruent. The correct answer is B.

Got It? **4.** Are $\triangle PAR$ and $\triangle SIR$ congruent? Explain.



Lesson Check

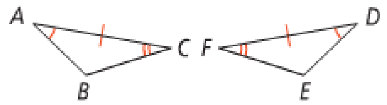
Do you know HOW?

1. In $\triangle RST$, which side is included between $\angle R$ and $\angle S$?

2. In $\triangle NOM$, $\overline{NO}$ is included between which angles?

Which postulate or theorem could you use to prove $\triangle ABC \cong \triangle DEF$?

3.



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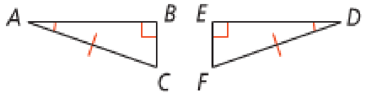
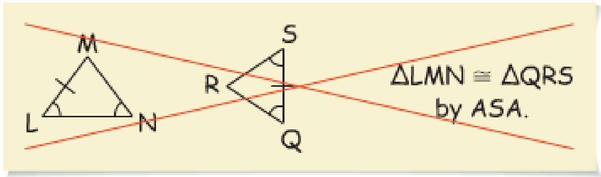


Do you UNDERSTAND?

5. Compare and Contrast How are the ASA Postulate and the SAS Postulate alike? How are they different?

6. Error Analysis Your friend asks you for help on a geometry exercise. Below is your friend's paper. What error did your friend make? Explain.



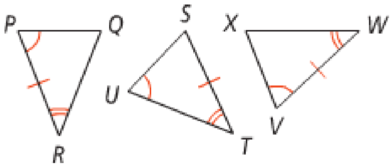
7. Reasoning Suppose $\angle E \cong \angle I$ and $\overline{FE} \cong \overline{GI}$. What else must you know in order to prove $\triangle FDE \cong \triangle GHI$ by ASA? By AAS?

Practice and Problem-Solving Exercises

A Practice

Name two triangles that are congruent by ASA. **See Problem 1.**

8.



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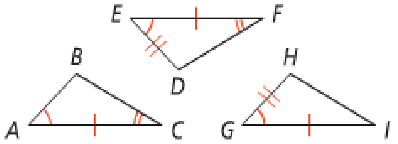
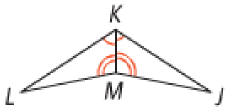


10. Developing Proof Complete the paragraph proof by filling in the blanks. **See Problem 2.**

Given: $\angle LKM \cong \angle JKM$,
$\angle LMK \cong \angle JMK$

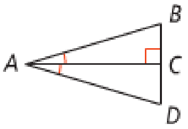
Prove: $\triangle LKM \cong \triangle JKM$



Proof: $\angle LKM \cong \angle JKM$ and $\angle LMK \cong \angle JMK$ are given. $\overline{KM} \cong \overline{KM}$ by the **a.** __?__ Property of Congruence. So, $\triangle LKM \cong \triangle JKM$ by **b.** __?__.

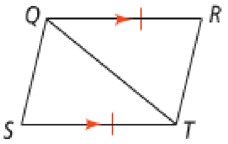
11. Proof Given: $\angle BAC \cong \angle DAC$,
$\overline{AC} \perp \overline{BD}$

Prove: $\triangle ABC \cong \triangle ADC$



12. Proof Given: $\overline{QR} \cong \overline{TS}$,
$\overline{QR} \parallel \overline{TS}$

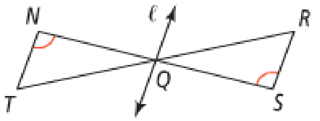
Prove: $\triangle QRT \cong \triangle TSQ$



13. **Developing Proof** Complete the two-column proof by filling in the blanks.

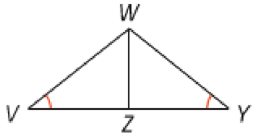
See Problem 3.

Given: $\angle N \cong \angle S$,
line ℓ bisects $\overline{TR}$ at Q
Prove: $\triangle NQT \cong \triangle SQR$

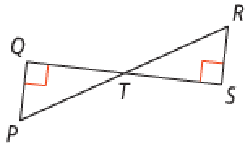


Statements	Reasons
1) $\angle N \cong \angle S$	1) Given
2) $\angle NQT \cong \angle SQR$	2) a. ?
3) Line ℓ bisects $\overline{TR}$ at Q.	3) b. ?
4) c. ?	4) Definition of bisect
5) $\triangle NQT \cong \triangle SQR$	5) d. ?

14. Proof **Given:** $\angle V \cong \angle Y$,
$\overline{WZ}$ bisects $\angle VWY$
Prove: $\triangle VWZ \cong \triangle YWZ$



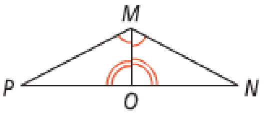
15. Proof **Given:** $\overline{PQ} \perp \overline{QS}$, $\overline{RS} \perp \overline{SQ}$,
T is the midpoint of $\overline{PR}$
Prove: $\triangle PQT \cong \triangle RST$



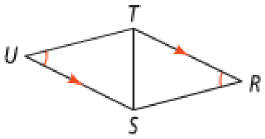
Determine whether the triangles must be congruent. If so, name the postulate or theorem that justifies your answer. If not, explain.

See Problem 4.

16.



17.



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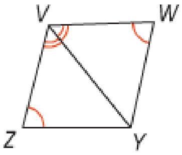
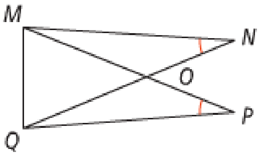
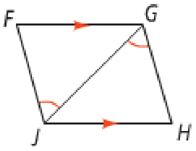


B Apply

19. Proof **Given:** $\angle N \cong \angle P$, $\overline{MO} \cong \overline{QO}$
Prove: $\triangle MON \cong \triangle QOP$

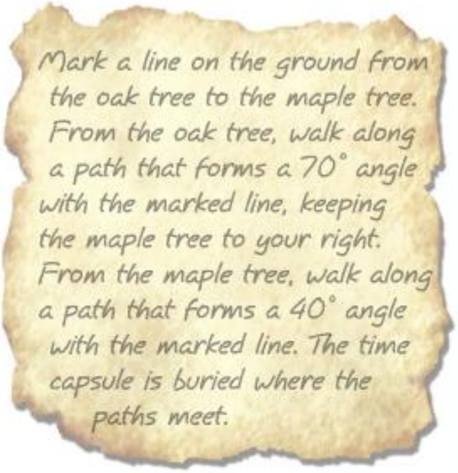


20. Proof **Given:** $\angle FJG \cong \angle HGJ$, $\overline{FG} \parallel \overline{JH}$
Prove: $\triangle FGJ \cong \triangle HJG$



21. **Think About a Plan** While helping your family clean out the attic, you find the piece of paper shown at the right. The paper contains clues to locate a time capsule buried in your backyard. The maple tree is due east of the oak tree in your backyard. Will the clues always lead you to the correct spot? Explain.

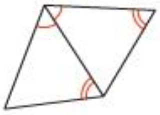
- How can you use a diagram to help you?
- What type of geometric figure do the paths and the marked line form?
- How does the position of the marked line relate to the positions of the angles?



Mark a line on the ground from the oak tree to the maple tree. From the oak tree, walk along a path that forms a 70° angle with the marked line, keeping the maple tree to your right. From the maple tree, walk along a path that forms a 40° angle with the marked line. The time capsule is buried where the paths meet.

22. **Constructions** Use a straightedge to draw a triangle. Label it $\triangle JKL$. Construct $\triangle MNP \cong \triangle JKL$ so that the triangles are congruent by ASA.

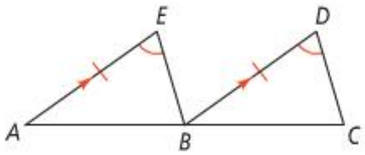
23. **Reasoning** Can you prove that the triangles at the right are congruent? Justify your answer.



24. **Writing** Anita says that you can rewrite any proof that uses the AAS Theorem as a proof that uses the ASA Postulate. Do you agree with Anita? Explain.

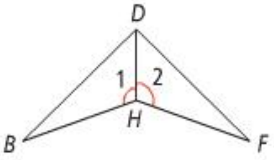
25. Proof **Given:** $\overline{AE} \parallel \overline{BD}$, $\overline{AE} \cong \overline{BD}$, $\angle E \cong \angle D$

Prove: $\triangle AEB \cong \triangle BDC$



26. Proof **Given:** $\angle 1 \cong \angle 2$, and $\overline{DH}$ bisects $\angle BDF$.

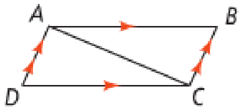
Prove: $\triangle BDH \cong \triangle FDH$



27. **Draw a Diagram** Draw two noncongruent triangles that have two pairs of congruent angles and one pair of congruent sides.

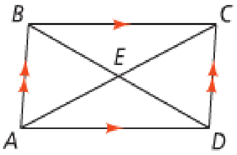
28. Proof **Given:** $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

Prove: $\triangle ABC \cong \triangle CDA$

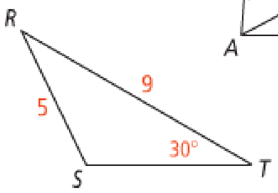


Challenge

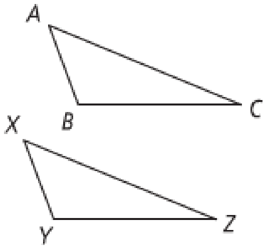
29. Given $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$, name as many pairs of congruent triangles as you can.



30. **Constructions** In $\triangle RST$ at the right, $RS = 5$, $RT = 9$, and $m\angle T = 30$. Show that there is no SSA congruence rule by constructing $\triangle UVW$ with $UV = RS$, $UW = RT$, and $m\angle W = m\angle T$, but with $\triangle UVW \ncong \triangle RST$.



31. **Probability** Below are six statements about the triangles at the right.



$\angle A \cong \angle X$ $\angle B \cong \angle Y$ $\angle C \cong \angle Z$

$\overline{AB} \cong \overline{XY}$ $\overline{AC} \cong \overline{XZ}$ $\overline{BC} \cong \overline{YZ}$

There are 20 ways to choose a group of three statements from these six. What is the probability that three statements chosen at random from the six will guarantee that the triangles are congruent?

Standardized Test Prep

SAT/ACT

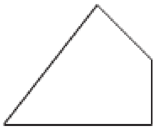
32. Suppose $\overline{RT} \cong \overline{ND}$ and $\angle R \cong \angle N$. What additional information do you need to prove that $\triangle RTJ \cong \triangle NDF$ by ASA?

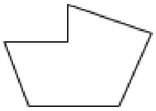
Ⓐ $\angle T \cong \angle D$ Ⓑ $\angle J \cong \angle F$ Ⓒ $\angle J \cong \angle D$ Ⓓ $\angle T \cong \angle F$

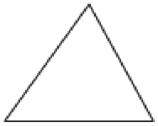
33. You plan to make a 2 ft-by-3 ft rectangular poster of class trip photos. Each photo is a 4 in.-by-6 in. rectangle. If the photos do not overlap, what is the greatest number of photos you can fit on your poster?

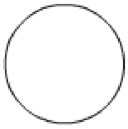
Ⓕ 4 Ⓖ 24 Ⓗ 32 Ⓘ 36

34. Which of the following figures is a concave polygon?

Ⓐ


Ⓑ


Ⓒ


Ⓓ


Short Response

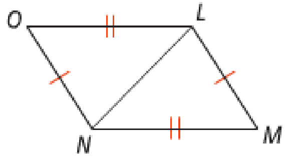
35. Write the converse of the true conditional statement below. Then determine whether the converse is true or false.

If you are less than 18 years old, then you are too young to vote in the United States.

Mixed Review

Would you use SSS or SAS to prove the triangles congruent? Explain. **See Lesson 4-2.**

36.



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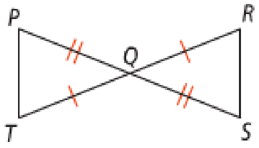


Get Ready! **To prepare for Lesson 4-4, do Exercises 38 and 39.**

For $\triangle TIC \cong \triangle LOK$, list the indicated parts. **See Lesson 4-1.**

38. congruent corresponding angles

39. congruent corresponding sides

Concept Byte
Use With Lesson 4-3
TECHNOLOGY

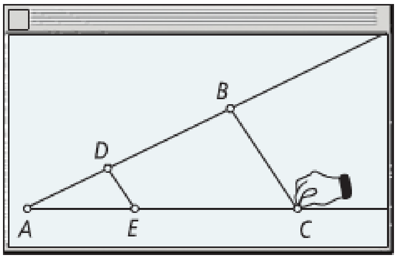
Exploring AAA and SSA

So far, you know four ways to conclude that two triangles are congruent—SSS, SAS, ASA, and AAS. It is good mathematics to wonder about the other two possibilities.

Activity 1

Construct Use geometry software to construct $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Construct $\overline{BC}$ to form $\triangle ABC$. Construct a line parallel to $\overline{BC}$ that intersects $\overrightarrow{AB}$ and $\overrightarrow{AC}$ at points D and E to form $\triangle ADE$.

Investigate Are the three angles of $\triangle ABC$ congruent to the three angles of $\triangle ADE$? Manipulate the figure to change the positions of $\overline{DE}$ and $\overline{BC}$. Do the corresponding angles of the triangles remain congruent? Are the two triangles congruent? Can the two triangles be congruent?

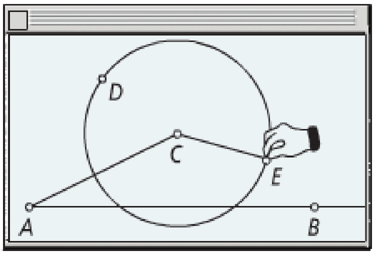


Activity 2

Construct Construct $\overrightarrow{AB}$. Draw a circle with center C that intersects $\overrightarrow{AB}$ at two points. Construct $\overline{AC}$. Construct point E on the circle and construct $\overline{CE}$.

Investigate Move point E around the circle until E is on $\overrightarrow{AB}$ and forms $\triangle ACE$. Then move E on the circle to the other point on $\overrightarrow{AB}$ to form another $\triangle ACE$.

Compare AC, CE, and $m\angle A$ in the two triangles. Are two sides and a nonincluded angle of one triangle congruent to two sides and a nonincluded angle of the other triangle? Are the triangles congruent? If you change the measure of $\angle A$ and the size of the circle, do you get the same results?



Exercises

1. **Make a Conjecture** Based on your first investigation above, can you prove triangles congruent using AAA? Explain.

For Exercises 2–4, use what you learned in your second investigation above.

2. **Make a Conjecture** Can you prove triangles congruent using SSA? Explain.
3. Manipulate the figure so that $\angle A$ is obtuse. Can the circle intersect $\overrightarrow{AB}$ twice to form two triangles? Would SSA work if the congruent angles are obtuse? Explain.
4. Suppose you are given $\overline{CE}$, $\overline{AC}$, and $\angle A$. What must be true about CE, AC, and $m\angle A$ so that you can construct exactly one $\triangle ACE$? (*Hint:* Consider cases.)

4 Mid-Chapter Quiz

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Do you know HOW?

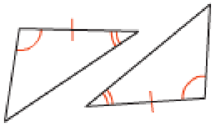
1. $\triangle RST \cong \triangle JKL$. List the three pairs of congruent corresponding sides and the three pairs of congruent corresponding angles.

$LMNO \cong PQRS$. Name the angle or side that corresponds to the given part.

2. $\overline{RS}$
3. $\angle L$
4. $\angle Q$
5. $\overline{MN}$

State the postulate or theorem you can use to prove the triangles congruent. If you do not have enough information to prove the triangles congruent, write *not enough information*.

6.



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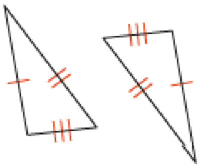

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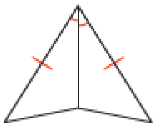

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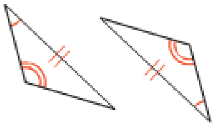

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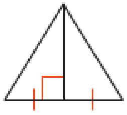

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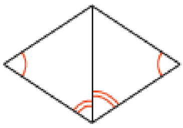
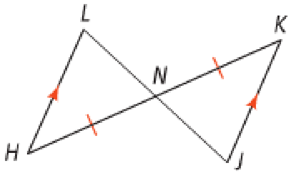


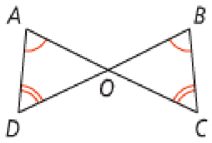
Use the diagram below. Tell why each statement is true.

12. $\angle H \cong \angle K$
13. $\angle HNL \cong \angle KNJ$
14. $\triangle HNL \cong \triangle KNJ$

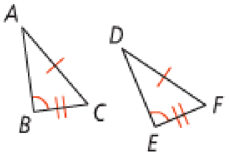


Determine what other information you need to prove the two triangles congruent. Then write the congruence statement and name the postulate or theorem you would use.

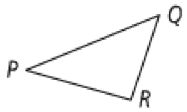
15.



16.

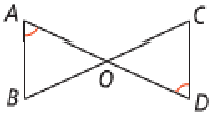


17. **Constructions** Construct $\triangle LMN$ congruent to $\triangle PQR$ using SSS.

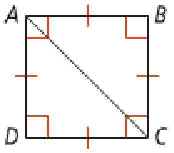


Do you UNDERSTAND?

18. **Given:** $\angle A \cong \angle D$, O is the midpoint of $\overline{AD}$
Prove: $\triangle AOB \cong \triangle DOC$



19. **Given:** $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$,
$\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles
Prove: $\triangle ABC \cong \triangle CDA$



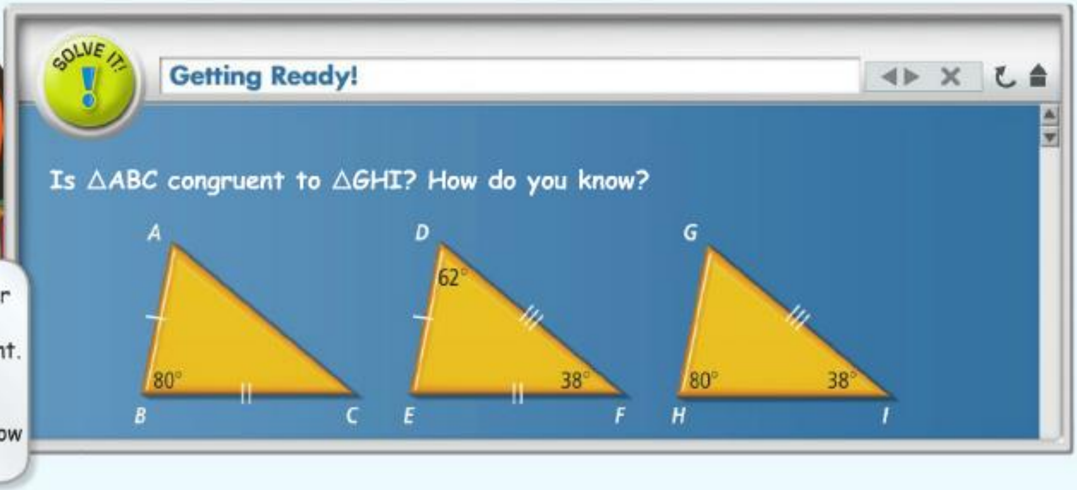
20. **Reasoning** Three segments form a triangle. How many unique triangles can you construct using the same three segments? Using the same three angles? Explain.

21. **Open-Ended** Write a congruency postulate for quadrilaterals. Does your postulate always hold true? Explain.

4-4 Using Corresponding Parts of Congruent Triangles

Objective To use triangle congruence and corresponding parts of congruent triangles to prove that parts of two triangles are congruent

Now you know four ways to prove triangles congruent. When you know triangles are congruent, you know a lot about them.



Is $\triangle ABC$ congruent to $\triangle GHI$? How do you know?

With SSS, SAS, ASA, and AAS, you know how to use three congruent parts of two triangles to show that the triangles are congruent. Once you know that two triangles are congruent, you can make conclusions about their other corresponding parts because, by definition, corresponding parts of congruent triangles are congruent.

Essential Understanding If you know two triangles are congruent, then you know that every pair of their corresponding parts is also congruent.

Think **Proof**

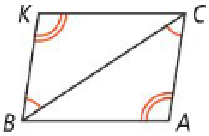
In the diagram, which congruent pair is not marked?

The third angles of both triangles are congruent. But there is no AAA congruence rule. So, find a congruent pair of sides.

Problem 1 Proving Parts of Triangles Congruent

Given: $\angle KBC \cong \angle ACB$, $\angle K \cong \angle A$

Prove: $\overline{KB} \cong \overline{AC}$



$\angle KBC \cong \angle ACB$
Given

$\overline{BC} \cong \overline{BC}$
Reflexive Property of ≅

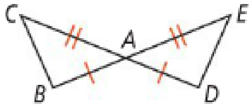
$\angle K \cong \angle A$
Given

$\triangle KBC \cong \triangle ACB$
AAS Theorem

$\overline{KB} \cong \overline{AC}$
Corresp. parts of ≅ △s are ≅.

Got It? **1. Given:** $\overline{BA} \cong \overline{DA}$, $\overline{CA} \cong \overline{EA}$

Prove: $\angle C \cong \angle E$



Proof

Problem 2 Proving Triangle Parts Congruent to Measure Distance

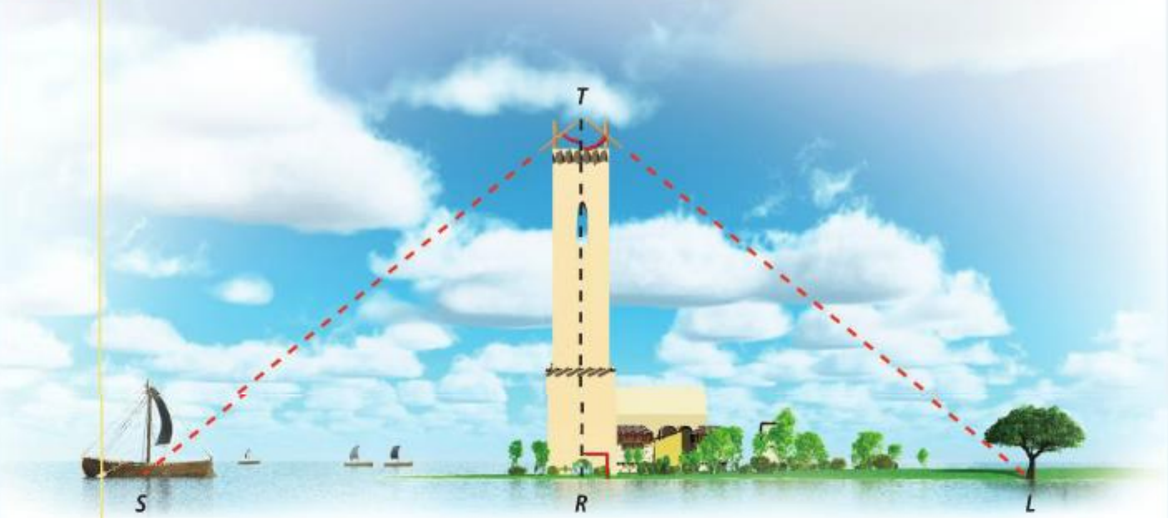
Measurement Thales, a Greek philosopher, is said to have developed a method to measure the distance to a ship at sea. He made a compass by nailing two sticks together. Standing on top of a tower, he would hold one stick vertical and tilt the other until he could see the ship S along the line of the tilted stick. With this compass setting, he would find a landmark L on the shore along the line of the tilted stick. How far would the ship be from the base of the tower?

Plan

Which congruency rule can you use? You have information about two pairs of angles. *Guess-and-check* AAS and ASA.

Given: $\angle TRS$ and $\angle TRL$ are right angles, $\angle RTS \cong \angle RTL$

Prove: $\overline{RS} \cong \overline{RL}$



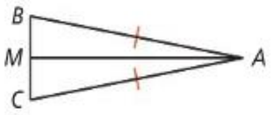
Statements	**Reasons**
1) $\angle RTS \cong \angle RTL$	**1)** Given
2) $\overline{TR} \cong \overline{TR}$	**2)** Reflexive Property of Congruence
3) $\angle TRS$ and $\angle TRL$ are right angles.	**3)** Given
4) $\angle TRS \cong \angle TRL$	**4)** All right angles are congruent.
5) $\triangle TRS \cong \triangle TRL$	**5)** ASA Postulate
6) $\overline{RS} \cong \overline{RL}$	**6)** Corresponding parts of $\cong$ $\triangle$s are $\cong$.

The distance between the ship and the base of the tower would be the same as the distance between the base of the tower and the landmark.

Got It? **2. a. Given:** $\overline{AB} \cong \overline{AC}$, M is the midpoint of $\overline{BC}$

Prove: $\angle AMB \cong \angle AMC$

b. Reasoning If the landmark were not at sea level, would the method in Problem 2 work? Explain.

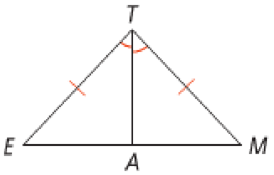


Lesson Check

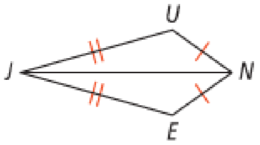
Do you know HOW?

Name the postulate or theorem that you can use to show the triangles are congruent. Then explain why the statement is true.

1. $\overline{EA} \cong \overline{MA}$



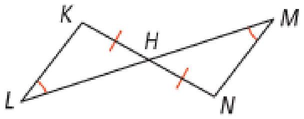
2. $\angle U \cong \angle E$



Do you UNDERSTAND?

3. Reasoning How does the fact that corresponding parts of congruent triangles are congruent relate to the definition of congruent triangles?

4. Error Analysis Find and correct the error(s) in the proof.



Given: $\overline{KH} \cong \overline{NH}$, $\angle L \cong \angle M$

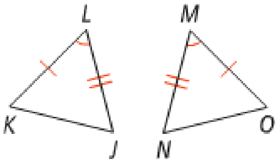
Prove: H is the midpoint of $\overline{LM}$.

Proof: $\overline{KH} \cong \overline{NH}$ because it is given. $\angle L \cong \angle M$ because it is given. $\angle KHL \cong \angle NHM$ because vertical angles are congruent. So, $\triangle KHL \cong \triangle MHN$ by ASA Postulate. Since corresponding parts of congruent triangles are congruent, $\overline{LH} \cong \overline{MH}$. By the definition of midpoint, H is the midpoint of $\overline{LM}$.

Practice and Problem-Solving Exercises

A Practice

5. Developing Proof Tell why the two triangles are congruent. Give the congruence statement. Then list all the other corresponding parts of the triangles that are congruent.

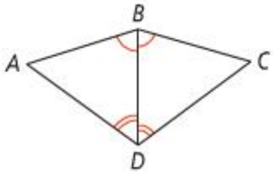


See Problem 1.

6. Given: $\angle ABD \cong \angle CBD$, $\angle BDA \cong \angle BDC$

Proof

Prove: $\overline{AB} \cong \overline{CB}$

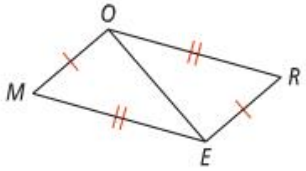


7. Given: $\overline{OM} \cong \overline{ER}$, $\overline{ME} \cong \overline{RO}$

Proof

Prove: $\angle M \cong \angle R$

See Problem 2.

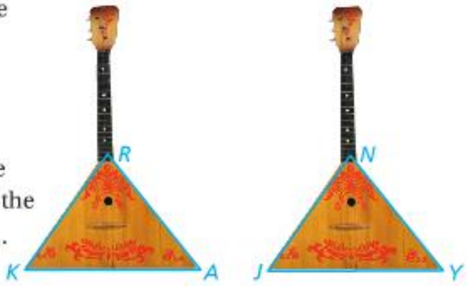


8. Developing Proof A balalaika is a stringed instrument. Prove that the bases of the balalaikas are congruent.

Given: $\overline{RA} \cong \overline{NY}$, $\angle KRA \cong \angle JNY$, $\angle KAR \cong \angle JYN$

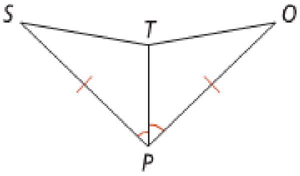
Prove: $\overline{KA} \cong \overline{JY}$

Proof: It is given that two angles and the included side of one triangle are congruent to two angles and the included side of the other. So, **a.** _?_ $\cong \triangle JNY$ by **b.** _?_. $\overline{KA} \cong \overline{JY}$ because **c.** _?_.

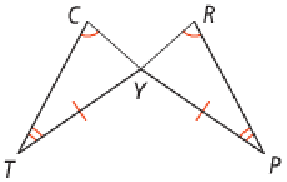


B Apply

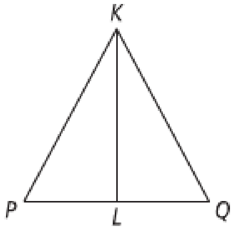
9. Given: $\angle SPT \cong \angle OPT$, $\overline{SP} \cong \overline{OP}$
Proof
Prove: $\angle S \cong \angle O$



10. Given: $\overline{YT} \cong \overline{YP}$, $\angle C \cong \angle R$, $\angle T \cong \angle P$
Proof
Prove: $\overline{CT} \cong \overline{RP}$



Reasoning **Copy and mark the figure to show the given information. Explain how you would prove $\angle P \cong \angle Q$.**



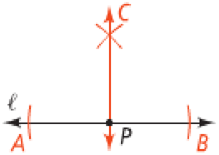
11. Given: $\overline{PK} \cong \overline{QK}$, $\overline{KL}$ bisects $\angle PKQ$

12. Given: $\overline{KL}$ is the perpendicular bisector of $\overline{PQ}$.

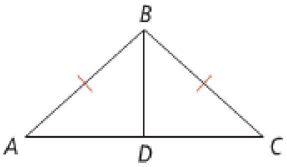
13. Given: $\overline{KL} \perp \overline{PQ}$, $\overline{KL}$ bisects $\angle PKQ$

14. Think About a Plan The construction of a line perpendicular to line ℓ through point P on line ℓ is shown. Explain why you can conclude that $\overleftrightarrow{CP}$ is perpendicular to ℓ.

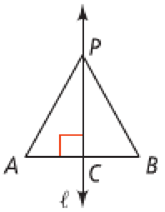
- How can you use congruent triangles to justify the construction?
- Which lengths or distances are equal by construction?



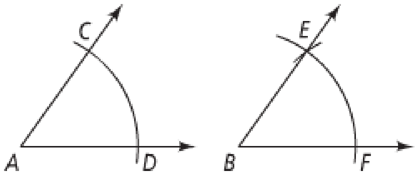
15. Given: $\overline{BA} \cong \overline{BC}$, $\overline{BD}$ bisects $\angle ABC$
Proof
Prove: $\overline{BD} \perp \overline{AC}$, $\overline{BD}$ bisects $\overline{AC}$



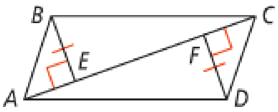
16. Given: $\ell \perp \overline{AB}$, ℓ bisects $\overline{AB}$ at C, P is on ℓ
Proof
Prove: $PA = PB$



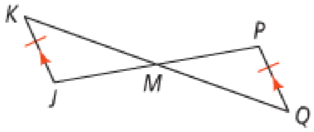
17. Constructions The construction of $\angle B$ congruent to given $\angle A$ is shown. $\overline{AD} \cong \overline{BF}$ because they are congruent radii. $\overline{DC} \cong \overline{FE}$ because both arcs have the same compass settings. Explain why you can conclude that $\angle A \cong \angle B$.



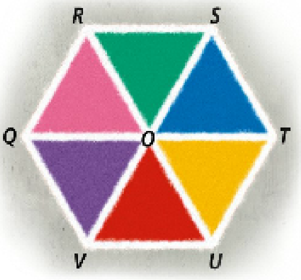
18. Given: $\overline{BE} \perp \overline{AC}$, $\overline{DF} \perp \overline{AC}$, $\overline{BE} \cong \overline{DF}$, $\overline{AF} \cong \overline{CE}$
Proof
Prove: $\overline{AB} \cong \overline{CD}$



19. Given: $\overline{JK} \parallel \overline{QP}$, $\overline{JK} \cong \overline{PQ}$
Proof
Prove: $\overline{KQ}$ bisects $\overline{JP}$.



20. **Designs** Rangoli is a colorful design pattern drawn outside houses in India, especially during festivals. Vina plans to use the pattern at the right as the base of her design. In this pattern, $\overline{RU}$, $\overline{SV}$, and $\overline{QT}$ bisect each other at O. $RS = 6$, $RU = 12$, $\overline{RU} \cong \overline{SV}$, $\overline{ST} \parallel \overline{RU}$, and $\overline{RS} \parallel \overline{QT}$. What is the perimeter of the hexagon?

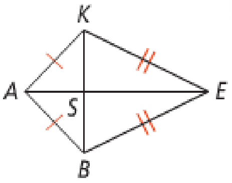


Challenge

In the diagram at the right, $\overline{BA} \cong \overline{KA}$ and $\overline{BE} \cong \overline{KE}$.

21. **Prove:** S is the midpoint of $\overline{BK}$.
Proof

22. **Prove:** $\overline{BK} \perp \overline{AE}$
Proof

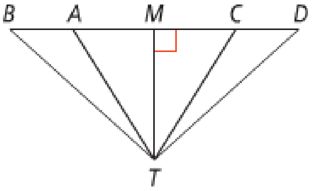


Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

For Exercises 23 and 24, use the diagram at the right. $\overline{TM} \perp \overline{BD}$ and $\overline{TM}$ bisects $\angle BTD$ and $\angle ATC$.



23. Suppose $BD = 17$ and $AM = 5$. What is the length of $\overline{CD}$?

24. Suppose $m\angle ATC = 64$, and $m\angle BTA = 16$. What is $m\angle B$?

25. Two parallel lines q and s are cut by a transversal t. $\angle 1$ and $\angle 2$ are a pair of alternate interior angles and $m\angle 2 = 38$. $\angle 1$ and $\angle 3$ are vertical angles. What is $m\angle 3$?

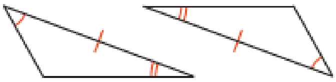
26. $\triangle ABC$ has vertices $A(1, 9)$, $B(4, 3)$, and $C(x, 6)$. For what value of x is $\triangle ABC$ a right triangle with right $\angle B$?

Mixed Review

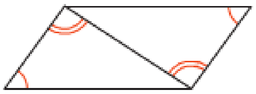
For Exercises 27 and 28, tell the postulate or theorem that you can use to prove the triangles congruent.

See Lesson 4-3.

27.



28.



Get Ready! **To prepare for Lesson 4-5, do Exercises 29–32.**

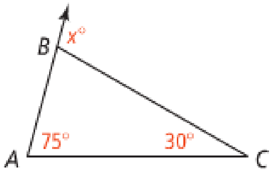
See Lesson 3-5.

29. What is the side opposite $\angle ABC$?

30. What is the angle opposite side $\overline{AB}$?

31. What is the angle opposite side $\overline{BC}$?

32. Find the value of x.



Concept Byte

Use With Lesson 4-5

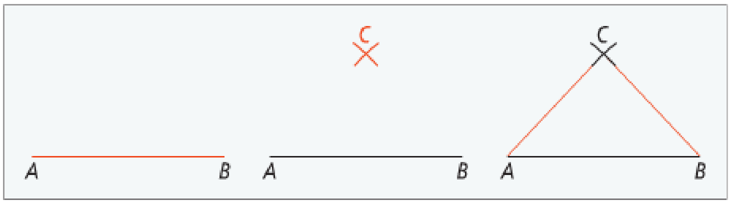
ACTIVITY

Paper-Folding Conjectures

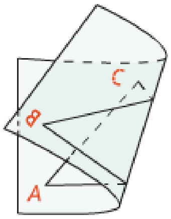
Isosceles triangles have two congruent sides. Folding one of the sides onto the other will suggest another important property of isosceles triangles.

Activity 1

Step 1 Construct an isosceles $\triangle ABC$ on tracing paper, with $\overline{AC} \cong \overline{BC}$.



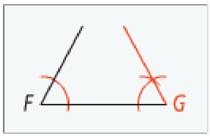
Step 2 Fold the paper so the two congruent sides fit exactly one on top of the other. Crease the paper. Label the intersection of the fold line and $\overline{AB}$ as point D.



1. What do you notice about $\angle A$ and $\angle B$? Compare your results with others. Make a conjecture about the angles opposite the congruent sides in an isosceles triangle.
2. **a.** Study the fold line $\overline{CD}$ and the base $\overline{AB}$. What type of angles are $\angle CDA$ and $\angle CDB$? How do $\overline{AD}$ and $\overline{BD}$ seem to be related?
   b. Use your answers to part (a) to complete the conjecture:
   The fold line $\overline{CD}$ is the $\underline{\;?\;}$ of the base $\overline{AB}$ of isosceles $\triangle ABC$.

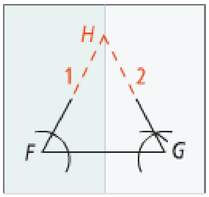
Activity 2

In Activity 1, you made a conjecture about angles opposite the congruent sides of a triangle. You can also fold paper to study whether the converse is true.



Step 1 On tracing paper, draw acute angle F and one side $\overline{FG}$. Construct $\angle G$ as shown, so that $\angle G \cong \angle F$.

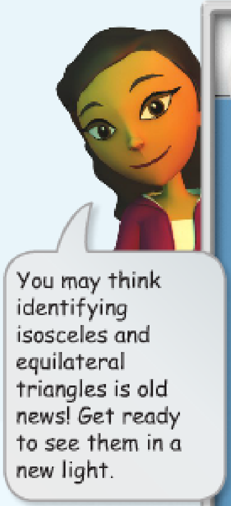
Step 2 Fold the paper so $\angle F$ and $\angle G$ fit exactly one on top of the other.

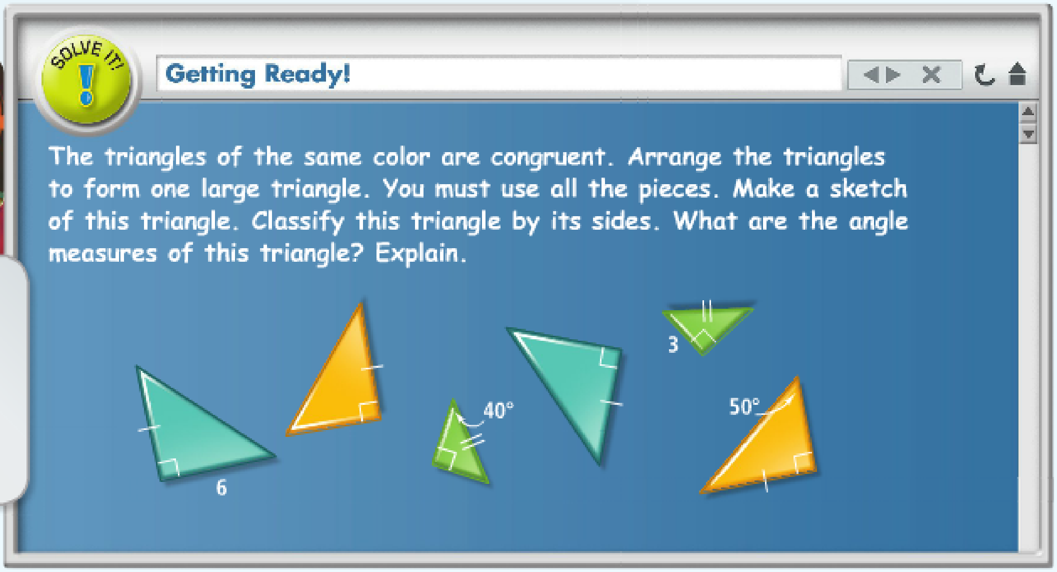


3. Why do sides 1 and 2 meet at point H on the fold line? Make a conjecture about sides $\overline{FH}$ and $\overline{GH}$ opposite congruent angles in a triangle.
4. Write your conjectures from Questions 1 and 3 as a biconditional.

4-5 Isosceles and Equilateral Triangles

Objective To use and apply properties of isosceles and equilateral triangles


You may think identifying isosceles and equilateral triangles is old news! Get ready to see them in a new light.



The triangles of the same color are congruent. Arrange the triangles to form one large triangle. You must use all the pieces. Make a sketch of this triangle. Classify this triangle by its sides. What are the angle measures of this triangle? Explain.

Dynamic Activity
Isosceles and Equilateral Triangles

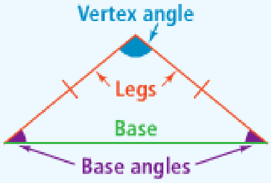
Lesson Vocabulary

- legs of an isosceles triangle
- base of an isosceles triangle
- vertex angle of an isosceles triangle
- base angles of an isosceles triangle
- corollary

In the Solve It, you classified a triangle based on the lengths of its sides. You can also identify certain triangles based on information about their angles. In this lesson, you will learn how to use and apply properties of isosceles and equilateral triangles.

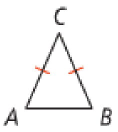
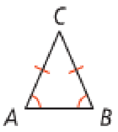
Essential Understanding The angles and sides of isosceles and equilateral triangles have special relationships.

Isosceles triangles are common in the real world. You can frequently see them in structures such as bridges and buildings, as well as in art and design. The congruent sides of an isosceles triangle are its **legs.** The third side is the **base.** The two congruent legs form the **vertex angle.** The other two angles are the **base angles.**



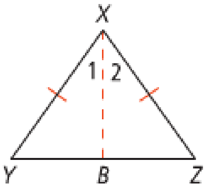
take note

Theorem 4-3 Isosceles Triangle Theorem

Theorem	If . . .	Then . . .
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.	$\overline{AC} \cong \overline{BC}$ 	$\angle A \cong \angle B$ 

The proof of the Isosceles Triangle Theorem requires an auxiliary line.

Proof Proof of Theorem 4-3: Isosceles Triangle Theorem



Begin with isosceles $\triangle XYZ$ with $\overline{XY} \cong \overline{XZ}$. Draw $\overline{XB}$, the bisector of the vertex angle $\angle YXZ$.

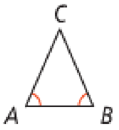
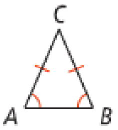
Given: $\overline{XY} \cong \overline{XZ}$, $\overline{XB}$ bisects $\angle YXZ$

Prove: $\angle Y \cong \angle Z$

Proof: $\overline{XY} \cong \overline{XZ}$ is given. By the definition of angle bisector, $\angle 1 \cong \angle 2$. By the Reflexive Property of Congruence, $\overline{XB} \cong \overline{XB}$. So by the SAS Postulate, $\triangle XYB \cong \triangle XZB$. $\angle Y \cong \angle Z$ since corresponding parts of congruent triangles are congruent.

take note

Theorem 4-4 Converse of the Isosceles Triangle Theorem

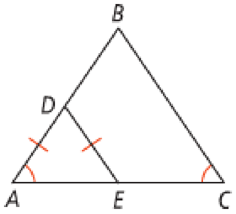
Theorem	If . . .	Then . . .
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.	$\angle A \cong \angle B$ 	$\overline{AC} \cong \overline{BC}$ 

You will prove Theorem 4-4 in Exercise 23.

Problem 1 Using the Isosceles Triangle Theorems

Think

What are you looking for in the diagram?
To use the Isosceles Triangle Theorems, you need a pair of congruent angles or a pair of congruent sides.

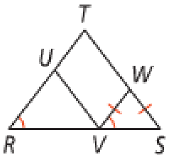


A Is $\overline{AB}$ congruent to $\overline{CB}$? Explain.

Yes. Since $\angle C \cong \angle A$, $\overline{AB} \cong \overline{CB}$ by the Converse of the Isosceles Triangle Theorem.

B Is $\angle A$ congruent to $\angle DEA$? Explain.

Yes. Since $\overline{AD} \cong \overline{ED}$, $\angle A \cong \angle DEA$ by the Isosceles Triangle Theorem.



Got It? **1. a.** Is $\angle WVS$ congruent to $\angle S$? Is $\overline{TR}$ congruent to $\overline{TS}$? Explain.

b. Reasoning Can you conclude that $\triangle RUV$ is isosceles? Explain.

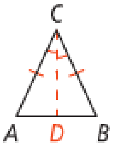
An isosceles triangle has a certain type of symmetry about a line through its vertex angle. The theorems in this lesson suggest this symmetry, which you will study in greater detail in Lesson 9-4.

take note

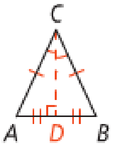
Theorem 4-5

Theorem
If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.

If . . .
$\overline{AC} \cong \overline{BC}$ and $\angle ACD \cong \angle BCD$



Then . . .
$\overline{CD} \perp \overline{AB}$ and $\overline{AD} \cong \overline{BD}$



You will prove Theorem 4-5 in Exercise 26.

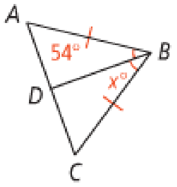
Think

What does the diagram tell you?
Since $\overline{AB} \cong \overline{CB}$, $\triangle ABC$ is isosceles. Since $\angle ABD \cong \angle CBD$, $\overline{BD}$ bisects the vertex angle of the isosceles triangle.

Problem 2 Using Algebra

GRIDDED RESPONSE

What is the value of x?



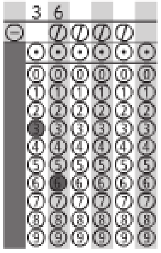
Since $\overline{AB} \cong \overline{CB}$, by the Isosceles Triangle Theorem, $\angle A \cong \angle C$. So $m\angle C = 54$.

Since $\overline{BD}$ bisects $\angle ABC$, you know by Theorem 4-5 that $\overline{BD} \perp \overline{AC}$. So $m\angle BDC = 90$.

$m\angle C + m\angle BDC + m\angle DBC = 180$ Triangle Angle-Sum Theorem

$54 + 90 + x = 180$ Substitute.

$x = 36$ Subtract 144 from each side.



Got It? **2.** Suppose $m\angle A = 27$. What is the value of x?

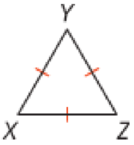
A **corollary** is a theorem that can be proved easily using another theorem. Since a corollary is a theorem, you can use it as a reason in a proof.

take note

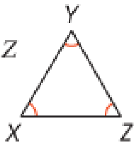
Corollary to Theorem 4-3

Corollary
If a triangle is equilateral, then the triangle is equiangular.

If . . .
$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



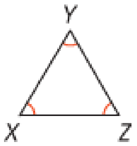
Then . . .
$\angle X \cong \angle Y \cong \angle Z$



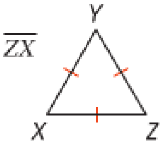
Corollary to Theorem 4-4

Corollary
If a triangle is equiangular, then the triangle is equilateral.

If . . .
$\angle X \cong \angle Y \cong \angle Z$

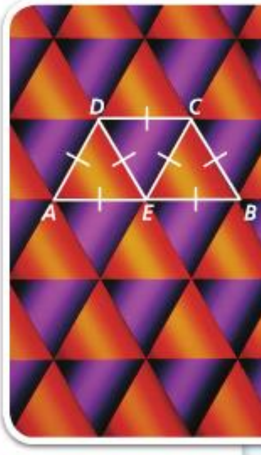


Then . . .
$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



Problem 3 Finding Angle Measures

Design What are the measures of $\angle A$, $\angle B$, and $\angle ADC$ in the photo at the right?



Think	Write
The triangles are equilateral, so they are also equiangular. Find the measure of each angle of an equilateral triangle.	Let a = measure of one angle. $3a = 180$ $a = 60$
$\angle A$ and $\angle B$ are both angles in an equilateral triangle.	$m\angle A = m\angle B = 60$
Use the Angle Addition Postulate to find the measure of $\angle ADC$.	$m\angle ADC = m\angle ADE + m\angle CDE$
Both $\angle ADE$ and $\angle CDE$ are angles in an equilateral triangle. So $m\angle ADE = 60$ and $m\angle CDE = 60$. Substitute into the above equation and simplify.	$m\angle ADC = 60 + 60$ $m\angle ADC = 120$

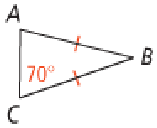
Got It? **3.** Suppose the triangles in Problem 3 are isosceles triangles, where $\angle ADE$, $\angle DEC$, and $\angle ECB$ are vertex angles. If the vertex angles each have a measure of 58, what are $m\angle A$ and $m\angle BCD$?

Lesson Check

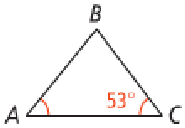
Do you know HOW?

1. What is $m\angle A$?

a.

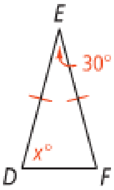


b.

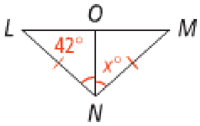


2. What is the value of x?

a.



b.



3. The measure of one base angle of an isosceles triangle is 23. What are the measures of the other two angles?

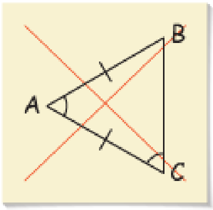
Do you UNDERSTAND?

4. What is the relationship between sides and angles for each type of triangle?

a. isosceles

b. equilateral

5. **Error Analysis** Claudia drew an isosceles triangle. She asked Sue to mark it. Explain why the marking of the diagram is incorrect.



Practice and Problem-Solving Exercises

A Practice

Complete each statement. Explain why it is true.

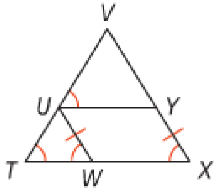
See Problem 1.

6. $\overline{VT} \cong \underline{\ ?\ }$

7. $\overline{UT} \cong \underline{\ ?\ } \cong \overline{YX}$

8. $\overline{VU} \cong \underline{\ ?\ }$

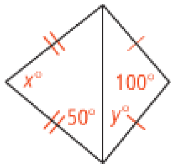
9. $\angle VYU \cong \underline{\ ?\ }$



Algebra **Find the values of *x* and *y*.**

See Problem 2.

10.



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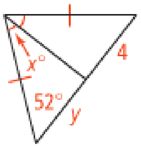
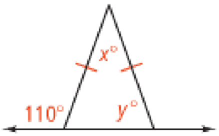
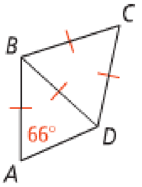


12.



13. An equilateral triangle and an isosceles triangle share a common side. What is the measure of $\angle ABC$?



See Problem 3.

B Apply

14. Architecture Each face of the Great Pyramid at Giza is an isosceles triangle with a 76° vertex angle. What are the measures of the base angles?

15. Reasoning What are the measures of the base angles of a right isosceles triangle? Explain.

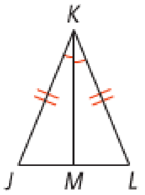
Given isosceles $\triangle JKL$ with base $\overline{JL}$, find each value.

16. If $m\angle L = 58$, then $m\angle LKJ = \underline{\ ?\ }$.

17. If $JL = 5$, then $ML = \underline{\ ?\ }$.

18. If $m\angle JKM = 48$, then $m\angle J = \underline{\ ?\ }$.

19. If $m\angle J = 55$, then $m\angle JKM = \underline{\ ?\ }$.

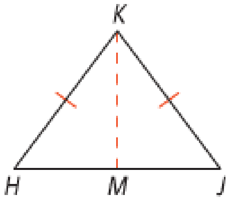


20. Think About a Plan A triangle has angle measures $x + 15$, $3x - 35$, and $4x$. What type of triangle is it? Be as specific as possible. Justify your answer.

- What do you know about the sum of the angle measures of a triangle?
- What do you need to know to classify a triangle?
- What type of triangle has no congruent angles? Two congruent angles? Three congruent angles?

21. Reasoning An exterior angle of an isosceles triangle has measure 100. Find two possible sets of measures for the angles of the triangle.

22. **Developing Proof** Here is another way to prove the Isosceles Triangle Theorem. Supply the missing information.



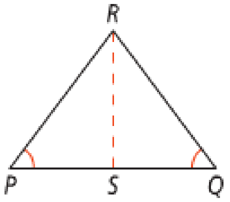
Begin with isosceles $\triangle HKJ$ with $\overline{KH} \cong \overline{KJ}$.
Draw **a.** $\underline{\ ?\ }$, a bisector of the base $\overline{HJ}$.

Given: $\overline{KH} \cong \overline{KJ}$, **b.** $\underline{\ ?\ }$ bisects $\overline{HJ}$

Prove: $\angle H \cong \angle J$

Statements	Reasons
1) $\overline{KM}$ bisects $\overline{HJ}$.	**1) c.** $\underline{\ ?\ }$
2) $\overline{HM} \cong \overline{JM}$	**2) d.** $\underline{\ ?\ }$
3) $\overline{KH} \cong \overline{KJ}$	**3)** Given
4) $\overline{KM} \cong \overline{KM}$	**4) e.** $\underline{\ ?\ }$
5) $\triangle KHM \cong \triangle KJM$	**5) f.** $\underline{\ ?\ }$
6) $\angle H \cong \angle J$	**6) g.** $\underline{\ ?\ }$

23. Supply the missing information in this statement of the Converse of the Isosceles
Proof Triangle Theorem. Then write a proof.



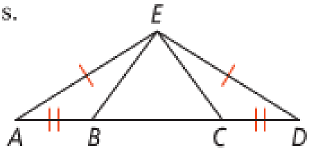
Begin with $\triangle PRQ$ with $\angle P \cong \angle Q$.
Draw **a.** $\underline{\ ?\ }$, the bisector of $\angle PRQ$.

Given: $\angle P \cong \angle Q$, **b.** $\underline{\ ?\ }$ bisects $\angle PRQ$

Prove: $\overline{PR} \cong \overline{QR}$

24. **Writing** Explain how the corollaries to the Isosceles Triangle Theorem and its converse follow from the theorems.

25. **Given:** $\overline{AE} \cong \overline{DE}$, $\overline{AB} \cong \overline{DC}$
Proof **Prove:** $\triangle ABE \cong \triangle DCE$

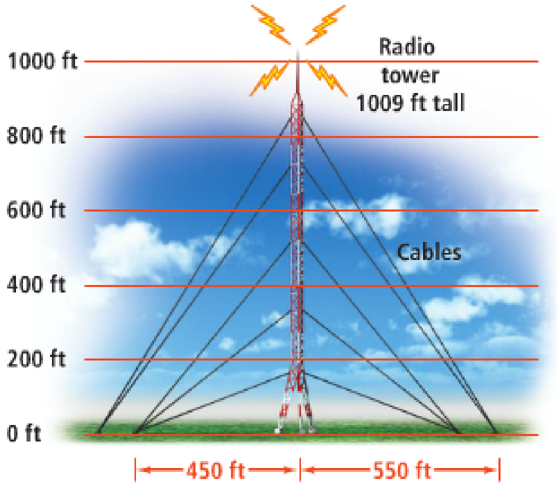


26. Prove Theorem 4-5. Use the diagram next to it on page 252.
Proof

27. **a.** **Communications** In the diagram at the right, what type of triangle is formed by the cables of the same height and the ground?

b. What are the two different base lengths of the triangles?

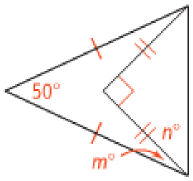
c. How is the tower related to each of the triangles?

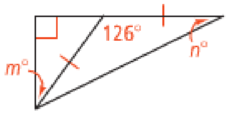


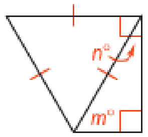
28. **Algebra** The length of the base of an isosceles triangle is x. The length of a leg is $2x - 5$. The perimeter of the triangle is 20. Find x.

29. **Constructions** Construct equiangular triangle ABC. Justify your method.

Algebra Find the values of m and n.

30. 

31. 

32. 

Challenge

Coordinate Geometry For each pair of points, there are six points that could be the third vertex of an isosceles right triangle. Find the coordinates of each point.

33. (4, 0) and (0, 4)

34. (0, 0) and (5, 5)

35. (2, 3) and (5, 6)

36. **Reasoning** What measures are possible for the base angles of an acute isosceles triangle?

Standardized Test Prep

SAT/ACT

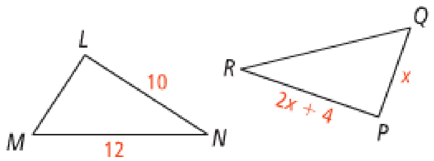
37. In isosceles $\triangle ABC$, the vertex angle is $\angle A$. What can you prove?

Ⓐ $AB = CB$ Ⓑ $m\angle B = m\angle C$ Ⓒ $\angle A \cong \angle B$ Ⓓ $\overline{BC} \cong \overline{AC}$

38. $\triangle LMN \cong \triangle PQR$. What is LM?

Ⓕ 3 Ⓗ 8

Ⓖ 4 Ⓘ 10



39. What is the exact area of the base of a circular swimming pool with diameter 16 ft?

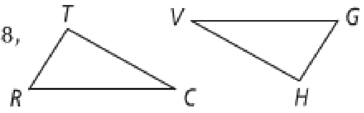
Ⓐ 1018.29 ft^2 Ⓑ 1018.3 ft^2 Ⓒ 64π ft^2 Ⓓ 256π ft^2

Short Response

40. Suppose $\triangle ABC$ and $\triangle DEF$ are nonright triangles. If $\angle B \cong \angle E$ and $\overline{AB} \cong \overline{DE}$, what else do you need to know to prove $\triangle ABC \cong \triangle DEF$? Explain.

Mixed Review

41. $m\angle R = 59$, $m\angle T = 93 = m\angle H$, $m\angle V = 28$, and $RT = GH$. What, if anything, can you conclude about RC and GV? Explain.



See Lesson 4-4.

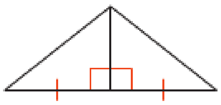
42. Find the pattern of the sequence M, T, W, T, F, . . . Then find the next two terms.

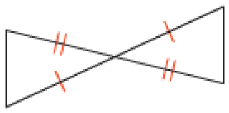
See Lesson 2-1.

Get Ready! **To prepare for Lesson 4-6, do Exercises 43 and 44.**

Can you conclude that the two triangles are congruent? Explain.

See Lesson 4-2.

43. 

44. 

Algebra Review

Use With Lesson 4-6

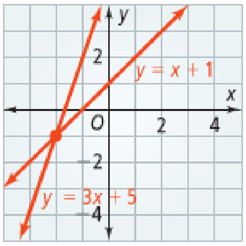
Systems of Linear Equations

You can solve a system of equations in two variables by using substitution.

Example 1

Algebra **Solve the system.**

$$\begin{aligned} y &= 3x + 5 \\ y &= x + 1 \end{aligned}$$



$$\begin{aligned} y &= x + 1 && \text{Start with one equation.} \\ 3x + 5 &= x + 1 && \text{Substitute } 3x + 5 \text{ for } y. \\ 2x &= -4 && \text{Solve for } x. \\ x &= -2 \end{aligned}$$

Substitute -2 for x in either equation and solve for y.

$y = x + 1 = (-2) + 1 = -1$

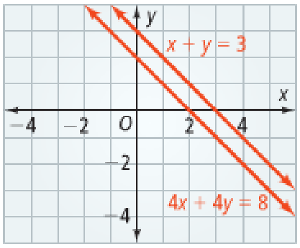
Since $x = -2$ and $y = 1$, the solution is $(-2, -1)$. This is the point of intersection of the two lines.

The graph of a linear system with *infinitely many solutions* is one line, and the graph of a linear system with *no solution* is two parallel lines.

Example 2

Algebra **Solve the system.**

$$\begin{aligned} x + y &= 3 \\ 4x + 4y &= 8 \end{aligned}$$



$$\begin{aligned} x + y &= 3 && \text{Start with one equation.} \\ x &= 3 - y && \text{Solve the equation for } x. \\ 4(3 - y) + 4y &= 8 && \text{Substitute } 3 - y \text{ for } x \text{ in the second equation.} \\ 12 - 4y + 4y &= 8 && \text{Solve for } y. \\ 12 &= 8 && \text{False!} \end{aligned}$$

Since $12 = 8$ is a false statement, the system has no solution.

Exercises

Solve each system of equations.

1. $y = x - 4$
$y = 3x + 2$

2. $2x - y = 8$
$x + 2y = 9$

3. $3x + y = 4$
$-6x - 2y = 12$

4. $2x - 3 = y + 3$
$2x + y = -3$

5. $y = x + 1$
$x = y - 1$

6. $x - y = 4$
$3x - 3y = 6$

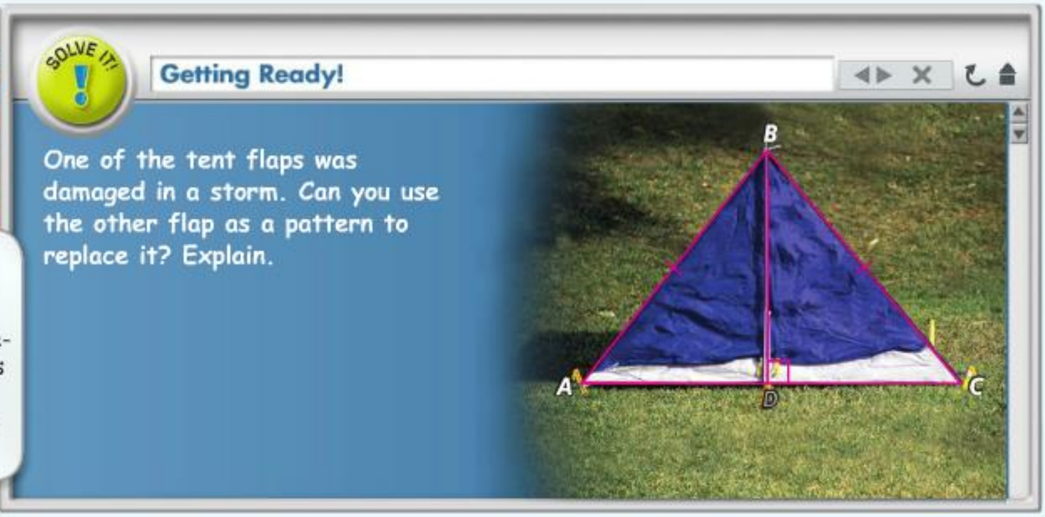
7. $y = -x + 2$
$2y = 4 - 2x$

8. $y = 2x - 1$
$y = 3x - 7$

4-6 Congruence in Right Triangles

Objective To prove right triangles congruent using the Hypotenuse-Leg Theorem

There isn't a lesson for proving triangles congruent by Side-Side-Angle. That's because it only works in the right situations.

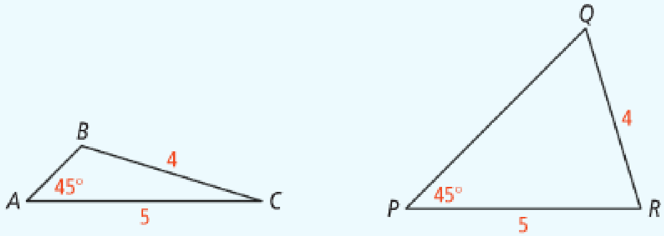


One of the tent flaps was damaged in a storm. Can you use the other flap as a pattern to replace it? Explain.

Dynamic Activity
Congruent Right Triangles

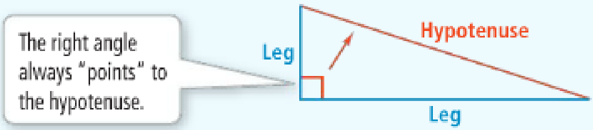
Lesson Vocabulary
- hypotenuse
- legs of a right triangle

In the diagram below, two sides and a nonincluded angle of one triangle are congruent to two sides and the nonincluded angle of another triangle.



Notice that the triangles are not congruent. So, you can conclude that Side-Side-Angle is *not* a valid method for proving two triangles congruent. This method, however, works in the special case of right triangles, where the right angles are the nonincluded angles.

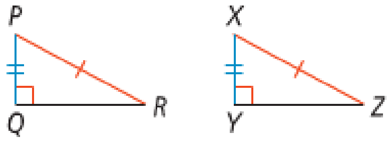
In a right triangle, the side opposite the right angle is called the **hypotenuse.** It is the longest side in the triangle. The other two sides are called **legs.**



Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove right triangles congruent by using one pair of right angles, a pair of hypotenuses, and a pair of legs.

take note

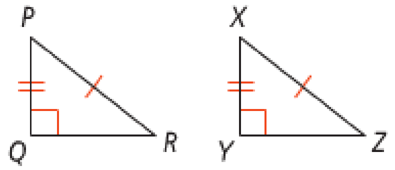
Theorem 4-6 Hypotenuse-Leg (HL) Theorem

Theorem	**If . . .**	**Then . . .**
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.	$\triangle PQR$ and $\triangle XYZ$ are right $\triangle$s, $\overline{PR} \cong \overline{XZ}$, and $\overline{PQ} \cong \overline{XY}$  	$\triangle PQR \cong \triangle XYZ$

To prove the HL Theorem you will need to draw auxiliary lines to make a third triangle.

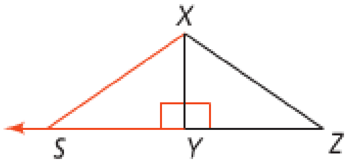
Proof

Proof of Theorem 4-6: Hypotenuse-Leg Theorem



Given: $\triangle PQR$ and $\triangle XYZ$ are right triangles, with right angles Q and Y. $\overline{PR} \cong \overline{XZ}$ and $\overline{PQ} \cong \overline{XY}$.

Prove: $\triangle PQR \cong \triangle XYZ$



Proof: On $\triangle XYZ$, draw $\overrightarrow{ZY}$.

Mark point S so that $YS = QR$. Then, $\triangle PQR \cong \triangle XYS$ by SAS.

Since corresponding parts of congruent triangles are congruent, $\overline{PR} \cong \overline{XS}$. It is given that $\overline{PR} \cong \overline{XZ}$, so $\overline{XS} \cong \overline{XZ}$ by the Transitive Property of Congruence. By the Isosceles Triangle Theorem, $\angle S \cong \angle Z$, so $\triangle XYS \cong \triangle XYZ$ by AAS. Therefore, $\triangle PQR \cong \triangle XYZ$ by the Transitive Property of Congruence.

take note

Key Concept Conditions for HL Theorem

To use the HL Theorem, the triangles must meet three conditions.

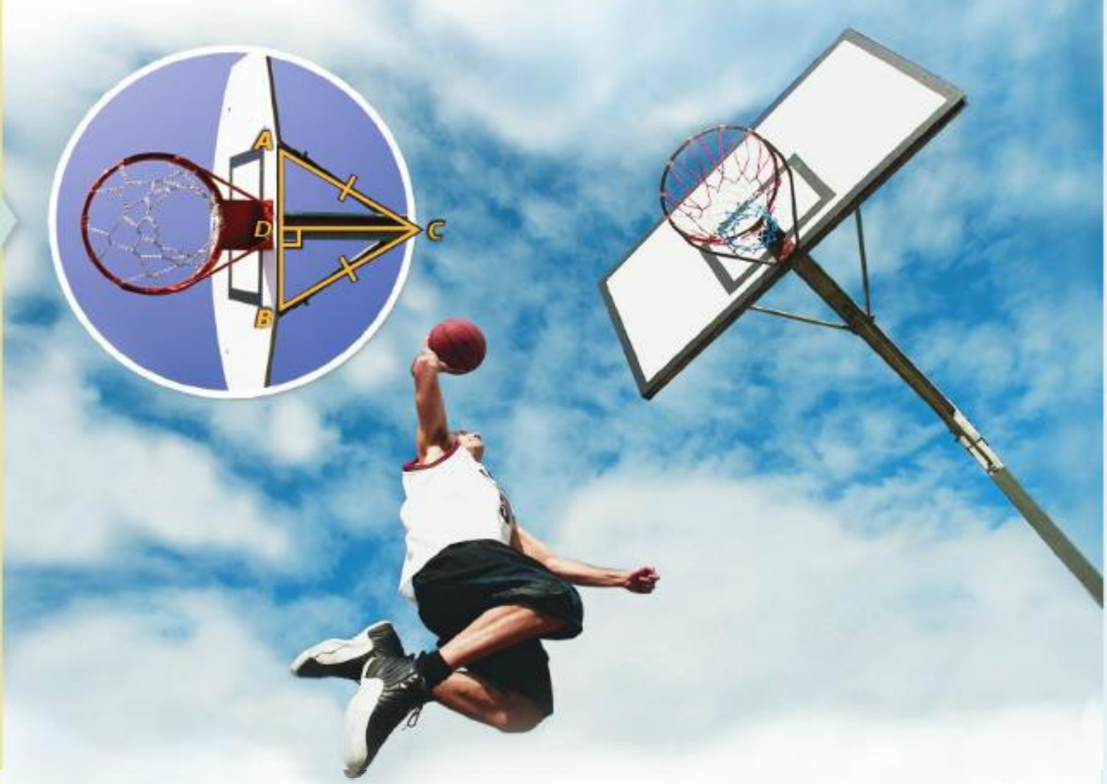
Conditions

- There are two right triangles.
- The triangles have congruent hypotenuses.
- There is one pair of congruent legs.

Proof

Problem 1 Using the HL Theorem

On the basketball backboard brackets shown below, $\angle ADC$ and $\angle BDC$ are right angles and $\overline{AC} \cong \overline{BC}$. Are $\triangle ADC$ and $\triangle BDC$ congruent? Explain.

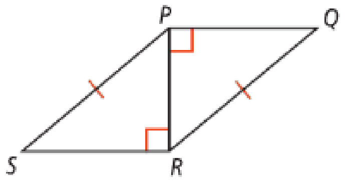


Plan

How can you visualize the two right triangles?
Imagine cutting $\triangle ABC$ along $\overline{DC}$. On either side of the cut, you get triangles with the same leg $\overline{DC}$.

- You are given that $\angle ADC$ and $\angle BDC$ are right angles. So, $\triangle ADC$ and $\triangle BDC$ are right triangles.
- The hypotenuses of the two right triangles are $\overline{AC}$ and $\overline{BC}$. You are given that $\overline{AC} \cong \overline{BC}$.
- $\overline{DC}$ is a common leg of both $\triangle ADC$ and $\triangle BDC$. $\overline{DC} \cong \overline{DC}$ by the Reflexive Property of Congruence.

Yes, $\triangle ADC \cong \triangle BDC$ by the HL Theorem.



Got It? **1. a. Given:** $\angle PRS$ and $\angle RPQ$ are right angles, $\overline{SP} \cong \overline{QR}$

Prove: $\triangle PRS \cong \triangle RPQ$

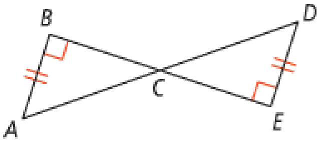
b. Reasoning Your friend says, "Suppose you have two right triangles with congruent hypotenuses and one pair of congruent legs. It does not matter which leg in the first triangle is congruent to which leg in the second triangle. The triangles will be congruent." Is your friend correct? Explain.

Proof

Problem 2 Writing a Proof Using the HL Theorem

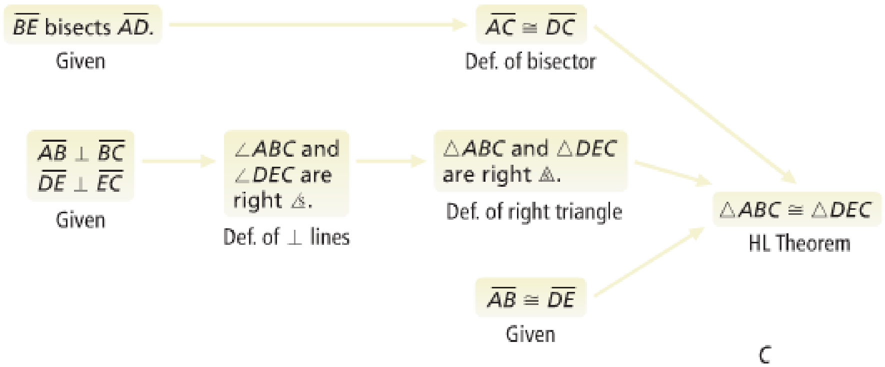
Plan

How can you get started?
Identify the hypotenuse of each right triangle. Prove that the hypotenuses are congruent.



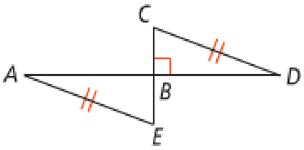
Given: $\overline{BE}$ bisects $\overline{AD}$ at C,
$\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EC}$, $\overline{AB} \cong \overline{DE}$

Prove: $\triangle ABC \cong \triangle DEC$


$\overline{BE}$ bisects $\overline{AD}$.
Given
$\overline{AC} \cong \overline{DC}$
Def. of bisector
$\overline{AB} \perp \overline{BC}$
$\overline{DE} \perp \overline{EC}$
Given
$\angle ABC$ and $\angle DEC$ are right $\triangle$s.
Def. of $\perp$ lines
$\triangle ABC$ and $\triangle DEC$ are right $\triangle$s.
Def. of right triangle
$\triangle ABC \cong \triangle DEC$
HL Theorem
$\overline{AB} \cong \overline{DE}$
Given

Got It? **2. Given:** $\overline{CD} \cong \overline{EA}$, $\overline{AD}$ is the perpendicular bisector of $\overline{CE}$

Prove: $\triangle CBD \cong \triangle EBA$

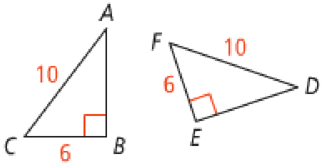


Lesson Check

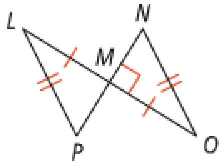
Do you know HOW?

Are the two triangles congruent? If so, write the congruence statement.

1.



2.



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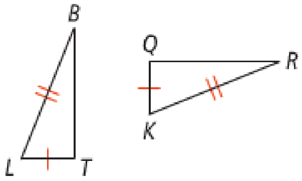
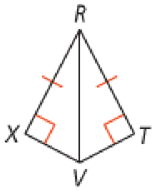


4.

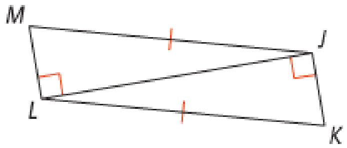


Do you UNDERSTAND?

5. Vocabulary A right triangle has side lengths of 5 cm, 12 cm, and 13 cm. What is the length of the hypotenuse? How do you know?

6. Compare and Contrast How do the HL Theorem and the SAS Postulate compare? How are they different? Explain.

7. Error Analysis Your classmate says that there is not enough information to determine whether the two triangles below are congruent. Is your classmate correct? Explain.



Practice and Problem-Solving Exercises

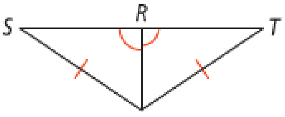
A Practice

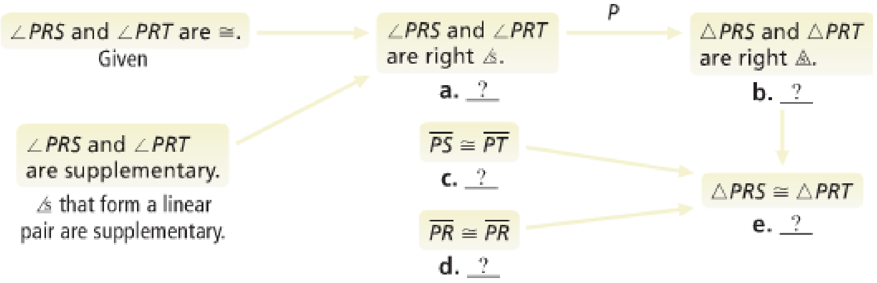
See Problem 1.

8. **Developing Proof** Complete the flow proof.

Given: $\overline{PS} \cong \overline{PT}$, $\angle PRS \cong \angle PRT$

Prove: $\triangle PRS \cong \triangle PRT$





$\angle PRS$ and $\angle PRT$ are $\cong$.
Given

$\angle PRS$ and $\angle PRT$ are right ∠s.
a. ?

$\triangle PRS$ and $\triangle PRT$ are right △s.
b. ?

$\angle PRS$ and $\angle PRT$ are supplementary.
∠s that form a linear pair are supplementary.

$\overline{PS} \cong \overline{PT}$
c. ?

$\overline{PR} \cong \overline{PR}$
d. ?

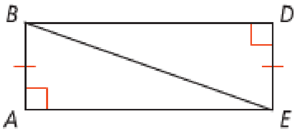
$\triangle PRS \cong \triangle PRT$
e. ?

9. **Developing Proof** Complete the paragraph proof.

Given: $\angle A$ and $\angle D$ are right angles, $\overline{AB} \cong \overline{DE}$

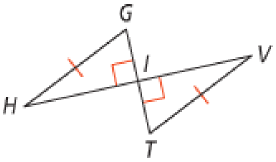
Prove: $\triangle ABE \cong \triangle DEB$

Proof: It is given that $\angle A$ and $\angle D$ are right angles. So, **a.** ? by the definition of right triangles. **b.** ?, because of the Reflexive Property of Congruence. It is also given that **c.** ?. So, $\triangle ABE \cong \triangle DEB$ by **d.** ?.



10. **Proof** **Given:** $\overline{HV} \perp \overline{GT}$, $\overline{GH} \cong \overline{TV}$, I is the midpoint of $\overline{HV}$

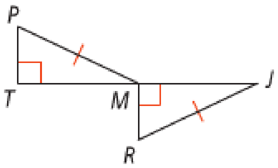
Prove: $\triangle IGH \cong \triangle ITV$



11. **Proof** **Given:** $\overline{PM} \cong \overline{RJ}$, $\overline{PT} \perp \overline{TJ}$, $\overline{RM} \perp \overline{TJ}$, M is the midpoint of $\overline{TJ}$

Prove: $\triangle PTM \cong \triangle RMJ$

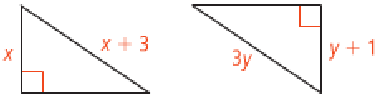
See Problem 2.



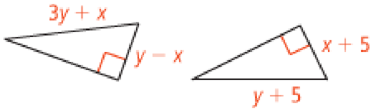
B Apply

Algebra **For what values of x and y are the triangles congruent by HL?**

12.



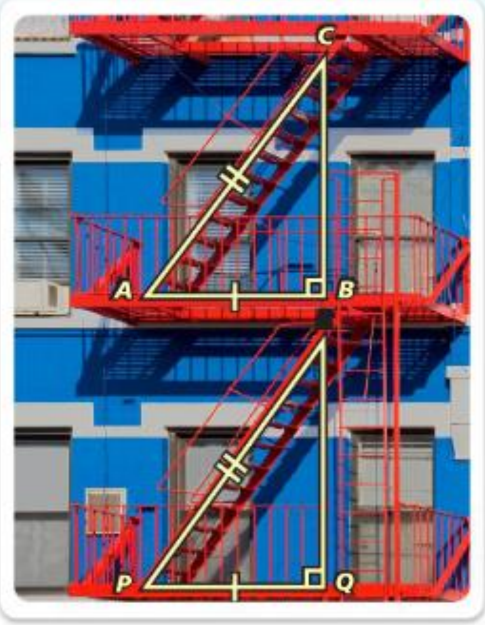
13.



14. Study Exercise 8. Can you prove that $\triangle PRS \cong \triangle PRT$ without using the HL Theorem? Explain.

15. Think About a Plan $\triangle ABC$ and $\triangle PQR$ are right triangular sections of a fire escape, as shown. Is each story of the building the same height? Explain.

- What can you tell from the diagram?
- How can you use congruent triangles here?

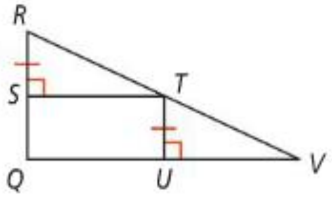


16. Writing "A HA!" exclaims your classmate. "There must be an HA Theorem, sort of like the HL Theorem!" Is your classmate correct? Explain.

17. Given: $\overline{RS} \cong \overline{TU}$, $\overline{RS} \perp \overline{ST}$, $\overline{TU} \perp \overline{UV}$, T is the midpoint of $\overline{RV}$

Proof

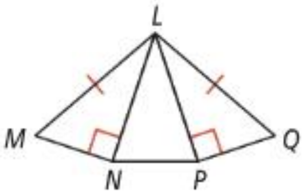
Prove: $\triangle RST \cong \triangle TUV$



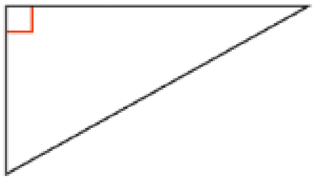
18. Given: $\triangle LNP$ is isosceles with base $\overline{NP}$, $\overline{MN} \perp \overline{NL}$, $\overline{QP} \perp \overline{PL}$, $\overline{ML} \cong \overline{QL}$

Proof

Prove: $\triangle MNL \cong \triangle QPL$



Constructions Copy the triangle and construct a triangle congruent to it using the given method.



19. SAS

20. HL

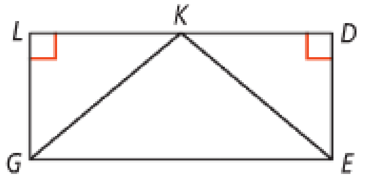
21. ASA

22. SSS

23. Given: $\triangle GKE$ is isosceles with base $\overline{GE}$, $\angle L$ and $\angle D$ are right angles, and K is the midpoint of $\overline{LD}$.

Proof

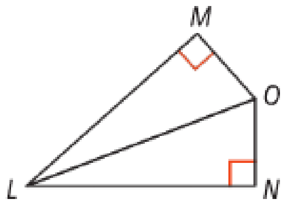
Prove: $\overline{LG} \cong \overline{DE}$



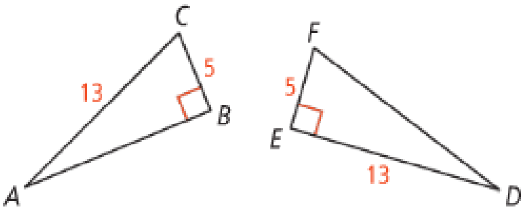
24. Given: $\overline{LO}$ bisects $\angle MLN$, $\overline{OM} \perp \overline{LM}$, $\overline{ON} \perp \overline{LN}$

Proof

Prove: $\triangle LMO \cong \triangle LNO$



25. Reasoning Are the triangles congruent? Explain.



26. a. **Coordinate Geometry** Graph the points $A(-5, 6)$, $B(1, 3)$, $D(-8, 0)$, and $E(-2, -3)$. Draw $\overline{AB}$, $\overline{AE}$, $\overline{BD}$, and $\overline{DE}$. Label point C, the intersection of $\overline{AE}$ and $\overline{BD}$.

b. Find the slopes of $\overline{AE}$ and $\overline{BD}$. How would you describe $\angle ACB$ and $\angle ECD$?

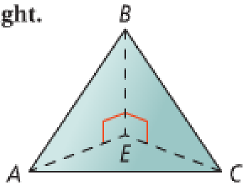
c. **Algebra** Write equations for $\overleftrightarrow{AE}$ and $\overleftrightarrow{BD}$. What are the coordinates of C?

d. Use the Distance Formula to find AB, BC, DC, and DE.

e. Write a paragraph to prove that $\triangle ABC \cong \triangle EDC$.

Challenge

Geometry in 3 Dimensions **For Exercises 27 and 28, use the figure at the right.**



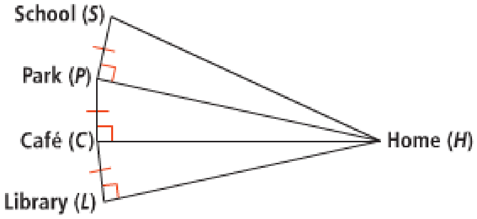
27. Given: $\overline{BE} \perp \overline{EA}$, $\overline{BE} \perp \overline{EC}$, $\triangle ABC$ is equilateral

Proof **Prove:** $\triangle AEB \cong \triangle CEB$

28. Given: $\triangle AEB \cong \triangle CEB$, $\overline{BE} \perp \overline{EA}$, $\overline{BE} \perp \overline{EC}$

Can you prove that $\triangle ABC$ is equilateral? Explain.

Standardized Test Prep



SAT/ACT

29. You often walk your dog around the neighborhood. Based on the diagram at the right, which of the following statements about distances is true?

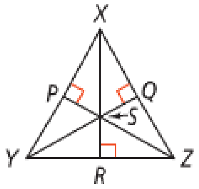
Ⓐ $SH = LH$ Ⓒ $SH > LH$

Ⓑ $PH = CH$ Ⓓ $PH < CH$

30. What is the midpoint of $\overline{LM}$ with endpoints $L(2, 7)$ and $M(5, -1)$?

Ⓕ (3.5, 3) Ⓗ (2, 4.5)

Ⓖ (3.5, 4) Ⓘ (7, 6)



Short Response

31. In equilateral $\triangle XYZ$, name four pairs of congruent right triangles. Explain why they are congruent.

Mixed Review

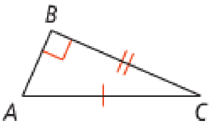
For Exercises 32 and 33, what type of triangle must $\triangle STU$ be? Explain. **See Lesson 4-5.**

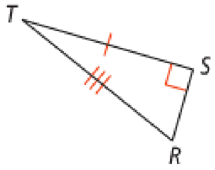
32. $\triangle STU \cong \triangle UTS$ **33.** $\triangle STU \cong \triangle UST$

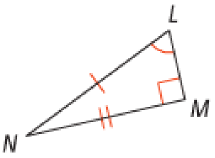
Get Ready! **To prepare for Lesson 4-7, do Exercises 34–36.**

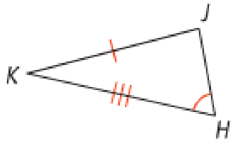
Can you conclude that the triangles are congruent? Explain. **See Lessons 4-3 and 4-6.**

34. $\triangle ABC$ and $\triangle LMN$ **35.** $\triangle LMN$ and $\triangle HJK$ **36.** $\triangle RST$ and $\triangle ABC$









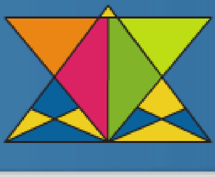
4-7 Congruence in Overlapping Triangles

Objectives To identify congruent overlapping triangles
To prove two triangles congruent using other congruent triangles

Do all the triangles make you dizzy? Try to see each one. Then learn some tricks that may help you.

SOLVE IT! **Getting Ready!**

An assignment for your graphic design class is to make a colorful design using triangles. How many triangles are in your design? Explain how you count them.



In the Solve It, you located individual triangles among a jumble of triangles. Some triangle relationships are difficult to see because the triangles overlap.

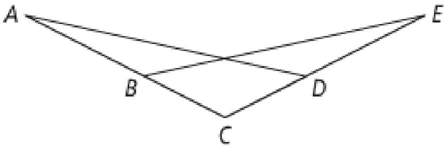
Essential Understanding You can sometimes use the congruent corresponding parts of one pair of congruent triangles to prove another pair of triangles congruent. This often involves overlapping triangles.

Overlapping triangles may have a common side or angle. You can simplify your work with overlapping triangles by separating and redrawing the triangles.

Think

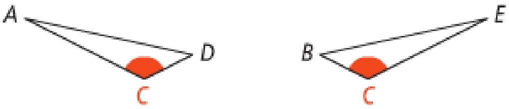
How can you see an individual triangle in order to redraw it?
Use your finger to trace along the lines connecting the three vertices. Then cover up any untraced lines.

Problem 1 Identifying Common Parts



What common angle do $\triangle ACD$ and $\triangle ECB$ share?

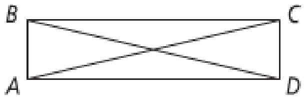
Separate and redraw $\triangle ACD$ and $\triangle ECB$.



The common angle is $\angle C$.

Got It? **1. a.** What is the common side in $\triangle ABD$ and $\triangle DCA$?

b. What is the common side in $\triangle ABD$ and $\triangle BAC$?

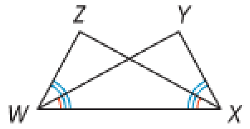


Proof

Problem 2 Using Common Parts

Given: $\angle ZXW \cong \angle YWX$, $\angle ZWX \cong \angle YXW$

Prove: $\overline{ZW} \cong \overline{YX}$

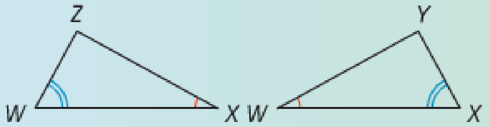


Know

- $\angle ZXW \cong \angle YWX$ and $\angle ZWX \cong \angle YXW$
- The diagram shows that $\triangle ZWX$ and $\triangle YXW$ are overlapping triangles.

Need

A diagram of the triangles separated



Plan

Show $\triangle ZWX \cong \triangle YXW$. Then use corresponding parts of congruent triangles to prove $\overline{ZW} \cong \overline{YX}$.

$\angle ZXW \cong \angle YWX$
Given

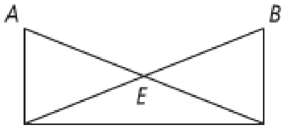
$\overline{WX} \cong \overline{WX}$
Reflexive Prop. of $\cong$

$\angle ZWX \cong \angle YXW$
Given

$\triangle ZWX \cong \triangle YXW$
ASA

$\overline{ZW} \cong \overline{YX}$
Corresp. parts of $\cong$ $\triangle$s are $\cong$.

Got It? **2. Given:** $\triangle ACD \cong \triangle BDC$
Prove: $\overline{CE} \cong \overline{DE}$



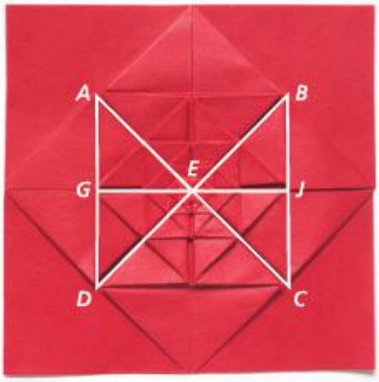
Proof

Problem 3 Using Two Pairs of Triangles

Given: In the origami design, E is the midpoint of $\overline{AC}$ and $\overline{DB}$.

Prove: $\triangle GED \cong \triangle JEB$

Proof: E is the midpoint of $\overline{AC}$ and $\overline{DB}$, so $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$. $\angle AED \cong \angle CEB$ because vertical angles are congruent. Therefore, $\triangle AED \cong \triangle CEB$ by SAS. $\angle D \cong \angle B$ because corresponding parts of congruent triangles are congruent. $\angle GED \cong \angle JEB$ because vertical angles are congruent. Therefore, $\triangle GED \cong \triangle JEB$ by ASA.

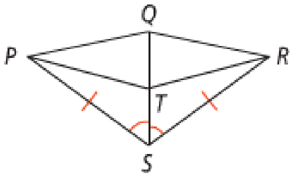


Plan

How do you choose another pair of triangles to help in your proof?
Look for triangles that share parts with $\triangle GED$ and $\triangle JEB$ and that you can prove congruent. In this case, first prove $\triangle AED \cong \triangle CEB$.

Got It? **3. Given:** $\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$

Prove: $\triangle QPT \cong \triangle QRT$



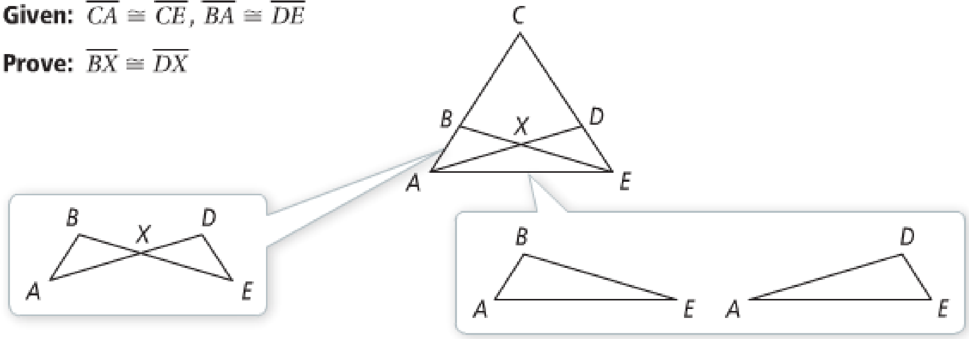
When several triangles overlap and you need to use one pair of congruent triangles to prove another pair congruent, you may find it helpful to draw a diagram of each pair of triangles.

Proof

Problem 4 Separating Overlapping Triangles

Given: $\overline{CA} \cong \overline{CE}$, $\overline{BA} \cong \overline{DE}$

Prove: $\overline{BX} \cong \overline{DX}$



Plan

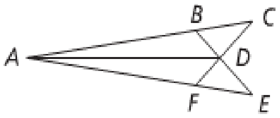
Which triangles are useful here?

If $\triangle BXA \cong \triangle DXE$, then $\overline{BX} \cong \overline{DX}$ because they are corresponding parts. If $\triangle BAE \cong \triangle DEA$, you will have enough information to show $\triangle BXA \cong \triangle DXE$.

Statements	Reasons
1) $\overline{BA} \cong \overline{DE}$	**1)** Given
2) $\overline{CA} \cong \overline{CE}$	**2)** Given
3) $\angle CAE \cong \angle CEA$	**3)** Base $\angle$s of an isosceles $\triangle$ are $\cong$.
4) $\overline{AE} \cong \overline{AE}$	**4)** Reflexive Property of $\cong$
5) $\triangle BAE \cong \triangle DEA$	**5)** SAS
6) $\angle ABE \cong \angle EDA$	**6)** Corresp. parts of $\cong$ $\triangle$s are $\cong$.
7) $\angle BXA \cong \angle DXE$	**7)** Vertical angles are $\cong$.
8) $\triangle BXA \cong \triangle DXE$	**8)** AAS
9) $\overline{BX} \cong \overline{DX}$	**9)** Corresp. parts of $\cong$ $\triangle$s are $\cong$.

Got It? **4. Given:** $\angle CAD \cong \angle EAD$, $\angle C \cong \angle E$

Prove: $\overline{BD} \cong \overline{FD}$

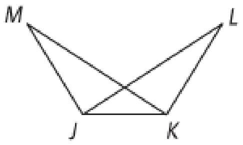


Lesson Check

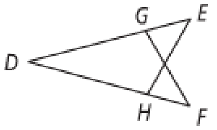
Do you know HOW?

Identify any common angles or sides.

1. $\triangle MKJ$ and $\triangle LJK$

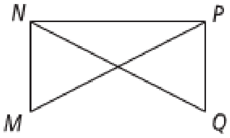


2. $\triangle DEH$ and $\triangle DFG$

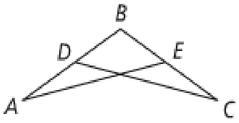


Separate and redraw the overlapping triangles. Label the vertices.

3.



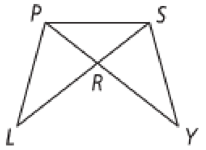
4.



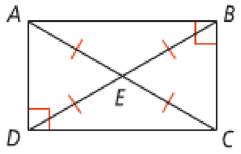
Do you UNDERSTAND?

5. Reasoning In Exercise 1, both triangles have vertices J and K. Are $\angle J$ and $\angle K$ common angles for $\triangle MKJ$ and $\triangle LJK$? Explain.

6. Error Analysis In the diagram, $\triangle PSY \cong \triangle SPL$. Based on that fact, your friend claims that $\triangle PRL \not\cong \triangle SRY$. Explain why your friend is incorrect.



7. In the figure below, which pair of triangles could you prove congruent first in order to prove that $\triangle ACD \cong \triangle CAB$? Explain.



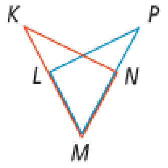
Practice and Problem-Solving Exercises

A Practice

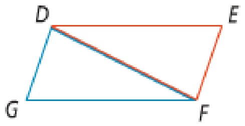
In each diagram, the red and blue triangles are congruent. Identify their common side or angle.

See Problem 1.

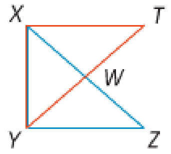
8.



9.

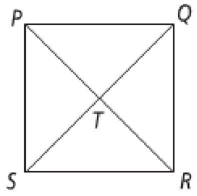


10.

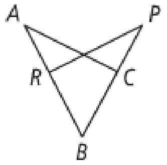


Separate and redraw the indicated triangles. Identify any common angles or sides.

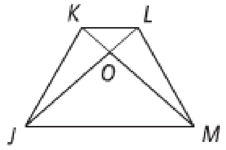
11. $\triangle PQS$ and $\triangle QPR$



12. $\triangle ACB$ and $\triangle PRB$



13. $\triangle JKL$ and $\triangle MLK$

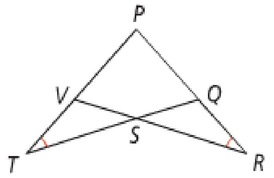


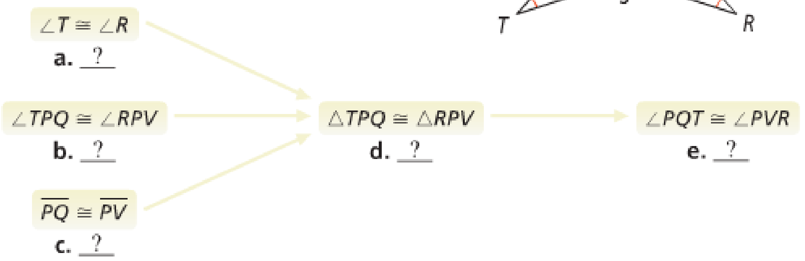
14. **Developing Proof** Complete the flow proof.

See Problem 2.

Given: $\angle T \cong \angle R$, $\overline{PQ} \cong \overline{PV}$

Prove: $\angle PQT \cong \angle PVR$





$\angle T \cong \angle R$
a. _?_

$\angle TPQ \cong \angle RPV$
b. _?_

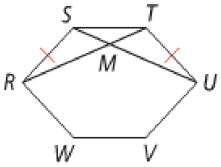
$\overline{PQ} \cong \overline{PV}$
c. _?_

$\triangle TPQ \cong \triangle RPV$
d. _?_

$\angle PQT \cong \angle PVR$
e. _?_

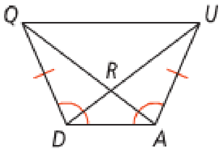
15. **Given:** $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

Proof **Prove:** $\triangle RST \cong \triangle UTS$



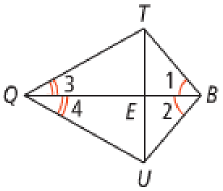
16. **Given:** $\overline{QD} \cong \overline{UA}$, $\angle QDA \cong \angle UAD$

Proof **Prove:** $\triangle QDA \cong \triangle UAD$



17. **Given:** $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Proof **Prove:** $\triangle QET \cong \triangle QEU$

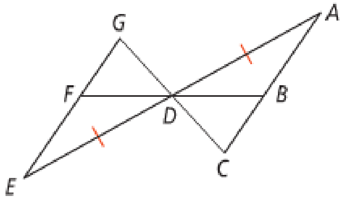


18. **Given:** $\overline{AD} \cong \overline{ED}$,

See Problems 3 and 4.

Proof D is the midpoint of $\overline{BF}$

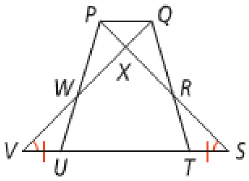
Prove: $\triangle ADC \cong \triangle EDG$



B Apply

19. **Think About a Plan** In the diagram at the right, $\angle V \cong \angle S$, $\overline{VU} \cong \overline{ST}$, and $\overline{PS} \cong \overline{QV}$. Which two triangles are congruent by SAS? Explain.

- How can you use a new diagram to help you identify the triangles?
- What do you need to prove triangles congruent by SAS?

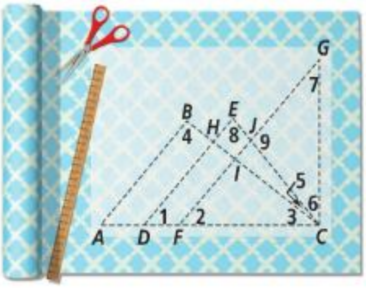


20. Clothing Design The figure at the right is part of a clothing design pattern, and it has the following relationships.

- $\overline{GC} \perp \overline{AC}$
- $\overline{AB} \perp \overline{BC}$
- $\overline{AB} \parallel \overline{DE} \parallel \overline{FG}$
- $m\angle A = 50$
- $\triangle DEC$ is isosceles with base $\overline{DC}$.

a. Find the measures of all the numbered angles in the figure.

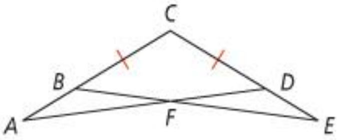
b. Suppose $\overline{AB} \cong \overline{FC}$. Name two congruent triangles and explain how you can prove them congruent.



21. Given: $\overline{AC} \cong \overline{EC}$, $\overline{CB} \cong \overline{CD}$

Proof

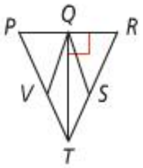
Prove: $\angle A \cong \angle E$



22. Given: $\overline{QT} \perp \overline{PR}$, $\overline{QT}$ bisects $\overline{PR}$, $\overline{QT}$ bisects $\angle VQS$

Proof

Prove: $\overline{VQ} \cong \overline{SQ}$



Open-Ended **Draw the diagram described.**

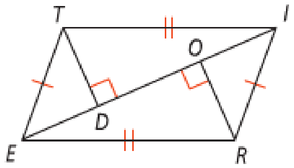
23. Draw a vertical segment on your paper. On the right side of the segment draw two triangles that share the vertical segment as a common side.

24. Draw two triangles that have a common angle.

25. Given: $\overline{TE} \cong \overline{RI}$, $\overline{TI} \cong \overline{RE}$, $\angle TDI$ and $\angle ROE$ are right ∡

Proof

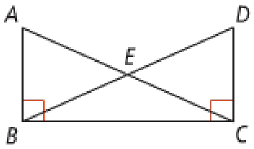
Prove: $\overline{TD} \cong \overline{RO}$



26. Given: $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$, $\overline{AC} \cong \overline{DB}$

Proof

Prove: $\overline{AE} \cong \overline{DE}$



Challenge

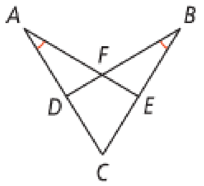
27. Reasoning Draw a quadrilateral $ABCD$ with $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$, and diagonals $\overline{AC}$ and $\overline{DB}$ intersecting at E. Label your diagram to indicate the parallel sides.

a. List all the pairs of congruent segments in your diagram.

b. **Writing** Explain how you know that the segments you listed are congruent.

28. **Proof** Identify a pair of overlapping congruent triangles in the diagram. Then use the given information to write a proof to show that the triangles are congruent.

Given: $\overline{AC} \cong \overline{BC}$, $\angle A \cong \angle B$



Standardized Test Prep

SAT/ACT

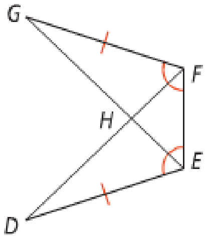
29. According to the diagram at the right, which statement is true?

(A) $\triangle DEH \cong \triangle GFH$ by AAS

(B) $\triangle DEH \cong \triangle GFH$ by SAS

(C) $\triangle DEF \cong \triangle GFE$ by AAS

(D) $\triangle DEF \cong \triangle GFE$ by SAS



30. $\triangle ABC$ is isosceles with base $\overline{AC}$. If $m\angle C = 37$, what is $m\angle B$?

(F) 37 (G) 74 (H) 106 (I) 143

31. Which word correctly completes the statement "All _?_ angles are congruent"?

(A) adjacent (B) supplementary (C) right (D) corresponding

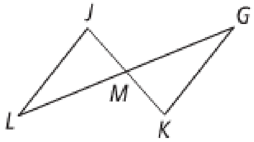
Extended Response

32. In the figure, $\overline{LJ} \parallel \overline{GK}$ and M is the midpoint of $\overline{LG}$.

a. Copy the diagram. Then mark your diagram with the given information.

b. Prove $\triangle LJM \cong \triangle GKM$.

c. Can you prove that $\triangle LJM \cong \triangle GKM$ another way? Explain.



Mixed Review

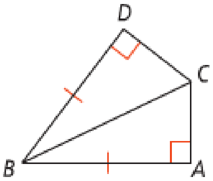
33. **Developing Proof** Complete the paragraph proof.

See Lesson 4-6.

Given: $\overline{AB} \cong \overline{DB}$, $\angle A$ and $\angle D$ are right angles

Prove: $\triangle ABC \cong \triangle DBC$

Proof: You are given that $\overline{AB} \cong \overline{DB}$ and $\angle A$ and $\angle D$ are right angles. $\triangle ABC$ and $\triangle DBC$ are **a.** _?_ triangles by the definition of **b.** _?_ triangle. $\overline{BC} \cong \overline{BC}$ by the **c.** _?_ Property of Congruence. $\triangle ABC \cong \triangle DBC$ by the **d.** _?_ Theorem.



34. **Constructions** Draw a line p and a point M not on p. Then construct line n through M so that $n \perp p$.

See Lesson 3-6.

Get Ready! **To prepare for Lesson 5-1, do Exercises 35–37.**

Find the coordinates of the midpoint of $\overline{AB}$.

See Lesson 1-7.

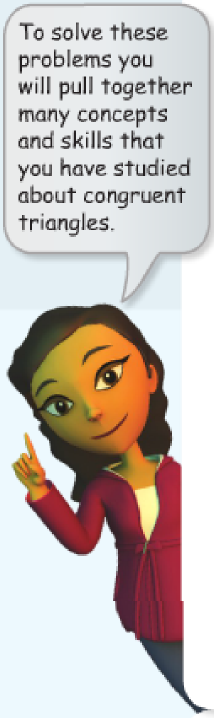
35. $A(-2, 3)$, $B(4, 1)$

36. $A(0, 5)$, $B(3, 6)$

37. $A(7, 10)$, $B(-5, -8)$

4

Pull It All Together

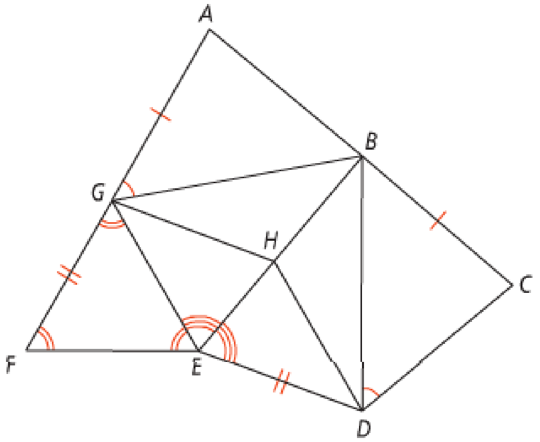

To solve these problems you will pull together many concepts and skills that you have studied about congruent triangles.

BIG idea Visualization

The corresponding parts of congruent triangles are the parts that match if you place the triangles on top of each other.

Task 1

Copy the diagram below. $\triangle GAB$ is isosceles with vertex angle A and $\triangle BCD$ is isosceles with vertex angle C. Is $\triangle BGH$ congruent to $\triangle BDH$? Justify your reasoning.



BIG idea Reasoning and Proof

You can prove triangles congruent if you know that certain relationships exist between corresponding parts. If you know that triangles are congruent, you know that all their corresponding parts are congruent.

Task 2

You and some neighbors are landscaping a community park. The organizer of the project selects an area for two congruent triangular rock gardens. You agree to be in charge of placing the pieces of wood to outline the gardens. The only tools you have are a saw, a protractor, and two very long pieces of wood. Describe one way to guarantee that the triangular outlines will be congruent. Justify your answer.

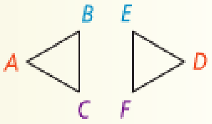
4 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Visualization
You can identify corresponding parts of congruent triangles by visualizing the figures placed on top of each other.

Congruent Figures (Lesson 4-1)

$\triangle ABC \cong \triangle DEF$

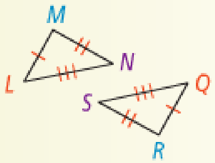


2 Reasoning and Proof
You can show two triangles are congruent by proving that certain relationships exist between three pairs of corresponding parts.

Proving Triangles Congruent (Lessons 4-2, 4-3, and 4-6)
Side-Side-Side (SSS), Side-Angle-Side (SAS), Angle-Side-Angle (ASA), Angle-Angle-Side (AAS), Hypotenuse-Leg (HL)

Using Corresponding Parts of Congruent Triangles (Lessons 4-4 and 4-7)

If $\triangle LMN \cong \triangle QRS$



$\overline{LM} \cong \overline{QR}$
$\overline{MN} \cong \overline{RS}$
$\overline{NL} \cong \overline{SQ}$
$\angle L \cong \angle Q$
$\angle M \cong \angle R$
$\angle N \cong \angle S$

3 Reasoning and Proof
You can tell whether a triangle is isosceles or equilateral by looking at the number of congruent angles or sides.

Isosceles and Equilateral Triangles (Lesson 4-5)
- The base angles of an isosceles triangle are congruent.
- All equilateral triangles are equiangular.
- All equiangular triangles are equilateral.

Chapter Vocabulary

- base angles of an isosceles triangle (p. 250)
- base of an isosceles triangle (p. 250)
- congruent polygons (p. 219)
- corollary (p. 252)
- hypotenuse (p. 258)
- legs of an isosceles triangle (p. 250)
- legs of a right triangle (p. 258)
- vertex angle of an isosceles triangle (p. 250)

Choose the correct term to complete each sentence.

1. The two congruent sides of an isosceles triangle are the ? .

2. The side opposite the right angle of a right triangle is the ? .

3. A ? to a theorem is a statement that follows immediately from the theorem.

4. ? have congruent corresponding parts.

4-1 Congruent Figures

Quick Review

Congruent polygons have congruent corresponding parts. When you name congruent polygons, always list corresponding vertices in the same order.

Example

$HIJK \cong PQRS$. Write all possible congruence statements.

The order of the parts in the congruence statement tells you which parts correspond.

Sides: $\overline{HI} \cong \overline{PQ}$, $\overline{IJ} \cong \overline{QR}$, $\overline{JK} \cong \overline{RS}$, $\overline{KH} \cong \overline{SP}$

Angles: $\angle H \cong \angle P$, $\angle I \cong \angle Q$, $\angle J \cong \angle R$, $\angle K \cong \angle S$

Exercises

$RSTUV \cong KLMNO$. Complete the congruence statements.

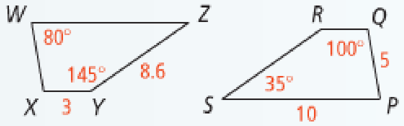
5. $\overline{TS} \cong \underline{\ ?\ }$

6. $\angle N \cong \underline{\ ?\ }$

7. $\overline{LM} \cong \underline{\ ?\ }$

8. $VUTSR \cong \underline{\ ?\ }$

$WXYZ \cong PQRS$. Find each measure or length.



9. $m\angle P$

10. QR

11. WX

12. $m\angle Z$

13. $m\angle X$

14. $m\angle R$

4-2 and 4-3 Triangle Congruence by SSS, SAS, ASA, and AAS

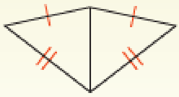
Quick Review

You can prove triangles congruent with limited information about their congruent sides and angles.

Postulate or Theorem	You need
Side-Side-Side (SSS)	three sides
Side-Angle-Side (SAS)	two sides and an included angle
Angle-Side-Angle (ASA)	two angles and an included side
Angle-Angle-Side (AAS)	two angles and a nonincluded side

Example

What postulate would you use to prove the triangles congruent?



You know that three sides are congruent. Use SSS.

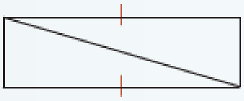
Exercises

15. In $\triangle HFD$, what angle is included between $\overline{DH}$ and $\overline{DF}$?

16. In $\triangle OMR$, what side is included between $\angle M$ and $\angle R$?

Which postulate or theorem, if any, could you use to prove the two triangles congruent? If there is not enough information to prove the triangles congruent, write *not enough information*.

17.



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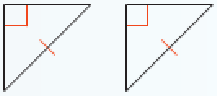

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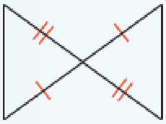

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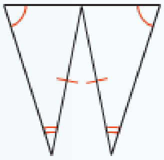


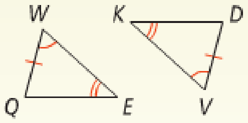
4-4 Using Corresponding Parts of Congruent Triangles

Quick Review

Once you know that triangles are congruent, you can make conclusions about corresponding sides and angles because, by definition, corresponding parts of congruent triangles are congruent. You can use congruent triangles in the proofs of many theorems.

Example

How can you use congruent triangles to prove $\angle Q \cong \angle D$?

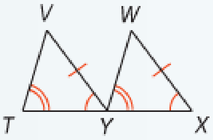


Since $\triangle QWE \cong \triangle DVK$ by AAS, you know that $\angle Q \cong \angle D$ because corresponding parts of congruent triangles are congruent.

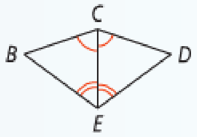
Exercises

How can you use congruent triangles to prove the statement true?

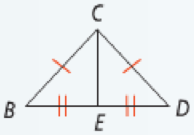
21. $\overline{TV} \cong \overline{YW}$



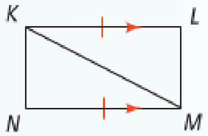
22. $\overline{BE} \cong \overline{DE}$



23. $\angle B \cong \angle D$



24. $\overline{KN} \cong \overline{ML}$



4-5 Isosceles and Equilateral Triangles

Quick Review

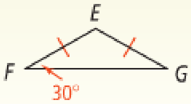
If two sides of a triangle are congruent, then the angles opposite those sides are also congruent by the **Isosceles Triangle Theorem.** If two angles of a triangle are congruent, then the sides opposite the angle are congruent by the **Converse of the Isosceles Triangle Theorem.**

Equilateral triangles are also equiangular.

Example

What is $m\angle G$?

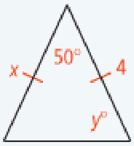
Since $\overline{EF} \cong \overline{EG}$, $\angle F \cong \angle G$ by the Isosceles Triangle Theorem. So $m\angle G = 30$.



Exercises

Algebra Find the values of x and y.

25.



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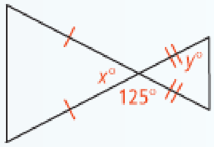

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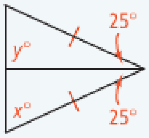

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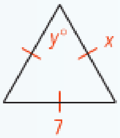


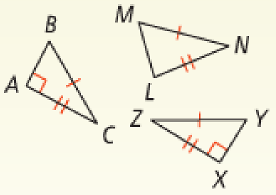
4-6 Congruence in Right Triangles

Quick Review

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent by the **Hypotenuse-Leg (HL) Theorem.**

Example

Which two triangles are congruent? Explain.



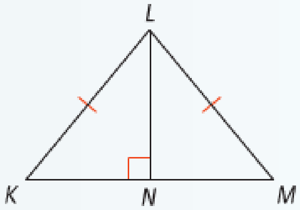
Since $\triangle ABC$ and $\triangle XYZ$ are right triangles with congruent legs, and $\overline{BC} \cong \overline{YZ}$, $\triangle ABC \cong \triangle XYZ$ by HL.

Exercises

Write a proof for each of the following.

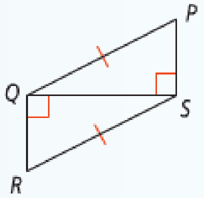
29. Given: $\overline{LN} \perp \overline{KM}$, $\overline{KL} \cong \overline{ML}$

Prove: $\triangle KLN \cong \triangle MLN$



30. Given: $\overline{PS} \perp \overline{SQ}$, $\overline{RQ} \perp \overline{QS}$, $\overline{PQ} \cong \overline{RS}$

Prove: $\triangle PSQ \cong \triangle RQS$



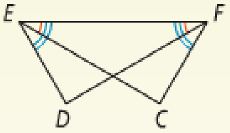
4-7 Congruence in Overlapping Triangles

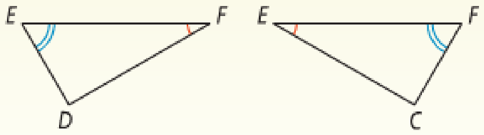
Quick Review

To prove overlapping triangles congruent, you look for the common or shared sides and angles.

Example

Separate and redraw the overlapping triangles. Label the vertices.

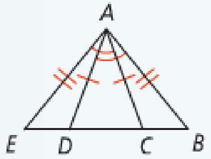




Exercises

Name a pair of overlapping congruent triangles in each diagram. State whether the triangles are congruent by SSS, SAS, ASA, AAS, or HL.

31.



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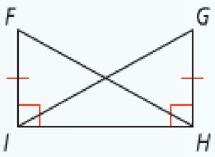

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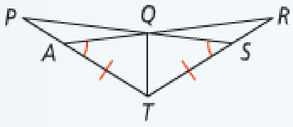

Chapter Test

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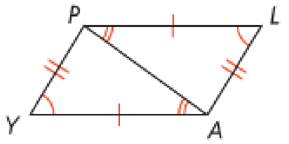
Do you know HOW?

Write a congruence statement for each pair of triangles.

1.



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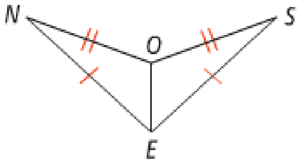
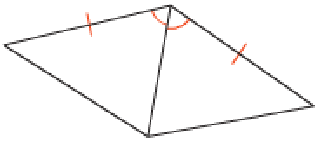


Which postulate or theorem, if any, could you use to prove the two triangles congruent? If not enough information is given, write *not enough information*.

3.



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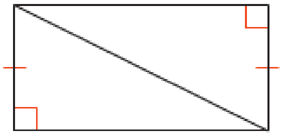

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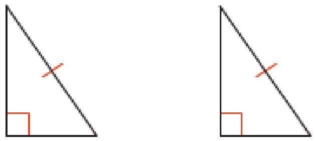

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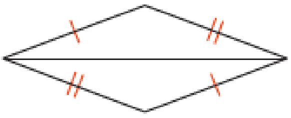

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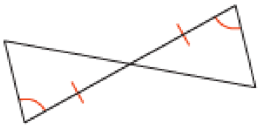
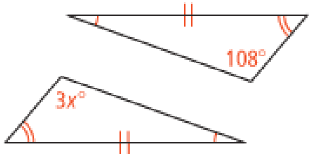


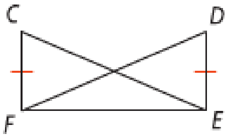
8. $\triangle CEO \cong \triangle HDF$. Name all of the pairs of corresponding congruent parts.

9. **Algebra** Find the value of x.

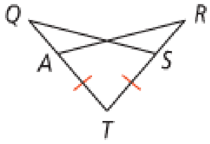


Name a pair of overlapping congruent triangles in each diagram. State whether the triangles are congruent by SSS, SAS, ASA, AAS, or HL.

10. **Given:** $\overline{CE} \cong \overline{DF}$, $\overline{CF} \cong \overline{DE}$



11. **Given:** $\overline{RT} \cong \overline{QT}$, $\overline{AT} \cong \overline{ST}$



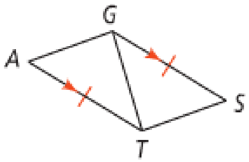
Do you UNDERSTAND?

12. **Reasoning** Isosceles $\triangle ABC$, with right $\angle B$, has a point D on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$. What is the relationship between $\triangle ABD$ and $\triangle CBD$? Explain.

Write a proof for each of the following.

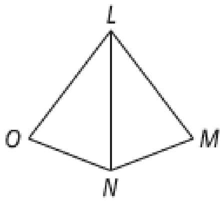
13. **Given:** $\overline{AT} \cong \overline{GS}$, $\overline{AT} \parallel \overline{GS}$

Prove: $\triangle GAT \cong \triangle TSG$



14. **Given:** $\overline{LN}$ bisects $\angle OLM$ and $\angle ONM$.

Prove: $\overline{ON} \cong \overline{MN}$



4 Cumulative Test Prep

TIPS FOR SUCCESS

Some test questions ask you to compare geometric figures. Read the sample question at the right. Then follow the tips to answer it.

TIP 2

You can use SSS to show triangles are congruent. Find the corresponding side lengths of $\triangle DEF$.

In the coordinate plane, the vertices of $\triangle ABC$ are $A(-2, 5)$, $B(2, 5)$, and $C(-2, 2)$. Which of the following are the side lengths for $\triangle DEF$, such that $\triangle ABC \cong \triangle DEF$?

Ⓐ $DE = 3, DF = 4, EF = 4$

Ⓑ $DE = 3, DF = 4, EF = 5$

Ⓒ $DE = 4, DF = 3, EF = 5$

Ⓓ $DE = 4, DF = 5, EF = 5$

TIP 1

Find the lengths of the sides of $\triangle ABC$.

Think It Through

Vertices A and B have the same y-coordinate and vertices A and C have the same x-coordinate, so $AB = |-2 - 2| = |-4| = 4$ and $AC = |5 - 2| = |3| = 3$.

By the Distance Formula, $BC = \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$. So $DE = AB = 4$, $DF = AC = 3$, and $EF = BC = 5$. The correct answer is C.

Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Match each term with its mathematical meaning.

A. slope

B. perpendicular lines

C. polygon

D. conjecture

E. congruent

I. lines that intersect to form right angles

II. having the same size and shape

III. a conclusion reached by inductive reasoning

IV. a closed plane figure with at least three sides that are segments

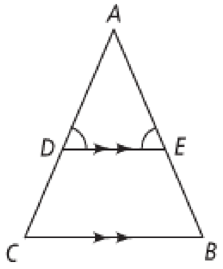
V. the ratio of the vertical change (rise) to the horizontal change (run)

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. Given: $\overline{DE} \parallel \overline{CB}$, $\angle ADE \cong \angle AED$

Prove: $\overline{AC} \cong \overline{AB}$



Proof: Since $\overline{DE} \parallel \overline{CB}$, $\angle ACB \cong \angle ADE$ and $\angle AED \cong \angle ABC$ by the Corresponding Angles Postulate. Since $\angle ADE \cong \angle AED$, $\angle ACB \cong \angle ABC$ by the Transitive Property. Which theorem or definition proves that $\overline{AC} \cong \overline{AB}$?

Ⓐ Isosceles Triangle Theorem

Ⓑ Converse of Isosceles Triangle Theorem

Ⓒ Alternate Interior Angles Theorem

Ⓓ Definition of congruent segments

2. Which statement must be true for two polygons to be congruent?

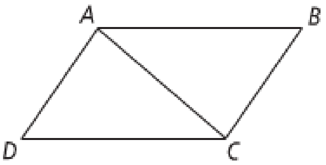
Ⓕ All the corresponding sides should be congruent.

Ⓖ All the corresponding sides and angles should be congruent.

Ⓗ All the corresponding angles should be congruent.

Ⓘ All sides in each polygon should be congruent.

3. If $\triangle ABC \cong \triangle CDA$, which of the following must be true?



Ⓐ $\overline{AB} \cong \overline{CA}$

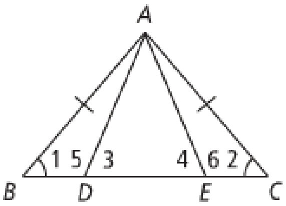
Ⓑ $\overline{BC} \cong \overline{DC}$

Ⓒ $\angle CAB \cong \angle ACD$

Ⓓ $\angle ABC \cong \angle CAD$

4. Given: $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{AC}$

What additional information do you need to prove $\triangle ABD \cong \triangle ACE$ by AAS?



Ⓕ $\overline{AD} \cong \overline{AE}$

Ⓖ $\overline{BD} \cong \overline{EC}$

Ⓗ $\angle 5 \cong \angle 6$

Ⓘ $\overline{BE} \cong \overline{DC}$

5. Which of the following statements is true?

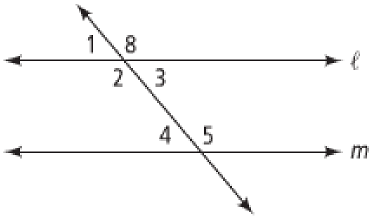
Ⓐ *Point, line,* and *plane* are undefined terms.

Ⓑ A theorem is an accepted statement of fact.

Ⓒ "Vertical angles are congruent" is a definition.

Ⓓ A postulate is a conjecture that is proven.

6. Which condition allows you to prove that $\ell \parallel m$?



Ⓕ $\angle 1 \cong \angle 8$

Ⓖ $\angle 2 \cong \angle 8$

Ⓗ $\angle 3 \cong \angle 4$

Ⓘ $\angle 3 \cong \angle 5$

7. A line passes through $(3, -4)$ and has a slope of -5. How can you find the y-intercept of the graph?

Ⓐ Substitute -5 for b, -4 for x, and 3 for y in $y = mx + b$. Then solve for m, the y-intercept.

Ⓑ Substitute -5 for b, 3 for x, and -4 for y in $y = mx + b$. Then solve for m, the y-intercept.

Ⓒ Substitute -5 for m, -4 for x, and 3 for y in $y = mx + b$. Then solve for b, the y-intercept.

Ⓓ Substitute -5 for m, 3 for x, and -4 for y in $y = mx + b$. Then solve for b, the y-intercept.

8. Given: $\triangle RST \cong \triangle LMN$

Which reason could you use to prove that $\angle R \cong \angle L$?

Ⓕ SAS

Ⓖ SSS

Ⓗ ASA

Ⓘ Corresponding parts of congruent triangles are congruent.

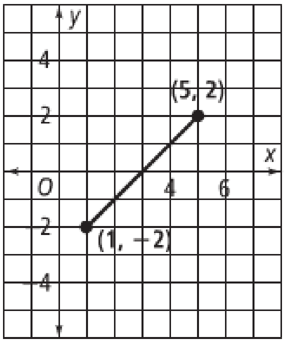
9. Which equation represents the perpendicular bisector of the segment shown?

Ⓐ $y = x - 3$

Ⓑ $3x + 3y = 3$

Ⓒ $y = 3x$

Ⓓ $x + y = 3$

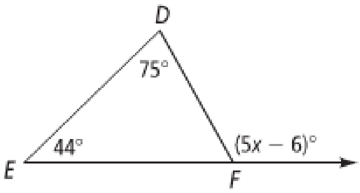


10. One bag of garden soil covers approximately 16 ft^2. If it takes 16π ft of fencing to enclose a circular garden, how many bags of soil do you need to cover the garden?

(F) 4 bags	(H) 13 bags
(G) 7 bags	(I) 16 bags

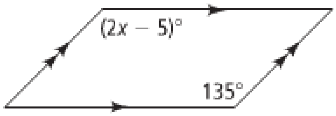
GRIDDED RESPONSE

11. What is the value of x in the figure below?

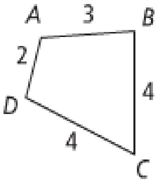


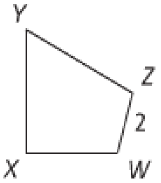
12. The length of a rectangle is seven more than three times its width. If the perimeter is 38 cm, what is the area, in square centimeters, of the rectangle?

13. What is the value of x in the figure below?

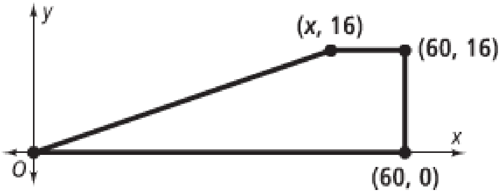


14. $ABCD \cong WXYZ$. What is WX?





15. Amy is designing a ramp up to a 16-in.-high skateboarding platform, as shown on the graph below. If she wants the slope of the ramp to be $\frac{1}{3}$, what value should she choose for the x-coordinate at the top of the ramp?



Short Response

16. Describe how the following everyday meanings of *acute* and *obtuse* help you to remember their mathematical meanings.

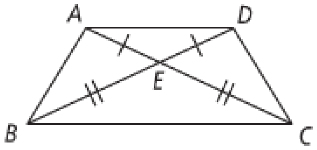
acute *adj.* Having a sharp point

obtuse *adj.* Not sharp or pointed; blunt

17. Write a proof for the following.

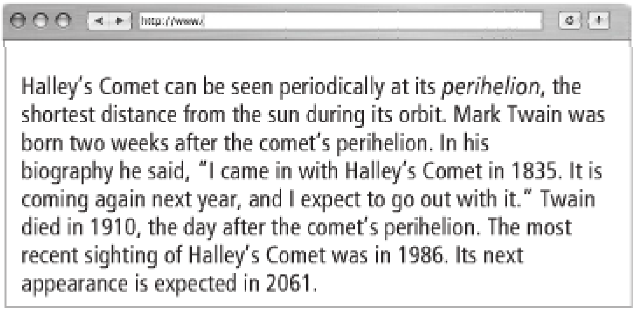
Given: $\overline{AE} \cong \overline{DE}$, $\overline{EB} \cong \overline{EC}$

Prove: $\triangle AEB \cong \triangle DEC$



Extended Response

18. Read this excerpt from an online news article.



Halley's Comet can be seen periodically at its *perihelion*, the shortest distance from the sun during its orbit. Mark Twain was born two weeks after the comet's perihelion. In his biography he said, "I came in with Halley's Comet in 1835. It is coming again next year, and I expect to go out with it." Twain died in 1910, the day after the comet's perihelion. The most recent sighting of Halley's Comet was in 1986. Its next appearance is expected in 2061.

a. Make a conjecture about the year in which Halley's Comet will appear after 2061. Explain your reasoning.

b. How confident are you about your conjecture? Explain.

19. The coordinates of the vertices of rectangle $LMNK$ are $L(-2, 5)$, $M(2, 5)$, $N(2, 3)$, and $K(-2, 3)$. The coordinates of the vertices of rectangle $PQRS$ are $P(3, 0)$, $Q(3, -3)$, $R(1, -3)$, and $S(1, 0)$. Are these two rectangles congruent? Explain why or why not. If not, how could you change the vertices of one of the rectangles to make them congruent?